

# NP Complete Problems-A Minimalist Mutatis Mutandis Model- Testament Of The Panoply

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## Abstract

A concatenation Model for the NP complete problems is given. Stability analysis, Solutional behavior are conducted. Due to space constraints, we do not go in to specification expatiations and enucleation of the diverse subjects and fields that the constituents belong to in the sense of widest commonality term.

## Introduction

NP Complete problems in physical reality comprise of

- (1) Soap Bubble
- (2) Protein Folding
- (3) Quantum Computing
- (4) Quantum Advice
- (5) Quantum Adiabatic algorithms
- (6) Quantum Mechanical Nonlinearities
- (7) Hidden Variables
- (8) Relativistic Time Dilation
- (9) Analog Computing
- (10) Malament-Hogarth Space Times
- (11) Quantum Gravity
- (12) Anthropic Computing

We give a minimalist concatenation model. We refer the reader to rich repository, receptacle, and reliquarium of literature available on the subject: Please note that the classification is done based on the physical parameters attributed and ascribed to the system or constituent in question with a comprehension of the concomitance of stratification in the other category. Any little intrusion into complex subjects would be egregiously presumptuous, an anathema and misnomer and will never do justice to the thematic and discursive form. Any attempt to give introductory remarks, essential predications, suspensional neutralities, rational representations, interfacial interference and syncopated justifications would only make the paper not less than 500 pages. We shall say that the P-NP problem itself is not solved let alone all the NP complete problems. We have taken a small step in this direction. More erudite scholars, we hope would take the insinuation made in the paper for further development and proliferation of the thesis propounded.

## Notation

### Soap Bubble And Protein Folding System: Variables Glossary

$G_{13}$  : Category One Of Soap Bubbles

$G_{14}$  : Category Two Of Soap Bubbles

$G_{15}$  : Category Three Of Soap Bubbles

$T_{13}$  : Category One Of Protein Folding

$T_{14}$  : Category Two Of Protein Folding

$T_{15}$  : Category Three Of Protein Folding

**Quantum Computing And Quantum Advice**

$G_{16}$  : Category One Quantum Computing

$G_{17}$  : Category Two Of Quantum Computing

$G_{18}$  : Category Three Of Quantum Computing

$T_{16}$  : Category One Of Quantum Advice

$T_{17}$  : Category Two Of Quantum Advice

$T_{18}$  : Category Three Of Quantum Advice

**Quantum Adiabatic Algorithms And Quantum Mechanical Nonlinearities**

$G_{20}$  : Category One Of Quantum Adiabatic Algorithms

$G_{21}$  : Category Two Of Quantum Adiabatic Algorithms

$G_{22}$  : Category Three Of Quantum Adiabatic Algorithms

$T_{20}$  : Category One Of Quantum Mechanical Nonlinearities

$T_{21}$  : Category Two Of Quantum Mechanical Nonlinearities

$T_{22}$  : Category Three Of Quantum Mechanical Nonlinearities

**Hidden Variables And Relativistic Time Dilation**

$G_{24}$  : Category One Of Hidden Variables

$G_{25}$  : Category Two Of Hidden Variables

$G_{26}$  : Category Three Of Hidden Variables

$T_{24}$  : Category One Of Relativistic Time Dilation

$T_{25}$  : Category Two Of Relativistic Time Dilation

$T_{26}$  : Category Three Of Relativistic Time Dilation

**Analog Computing And Malament Hogarth Space Times**

$G_{28}$  : Category One Of Analog Computing

$G_{29}$  : Category Two Of Analog Computing

$G_{30}$  : Category Three Of Analog Computing

$T_{28}$  : Category One Of Malament Hogarth Space Times

$T_{29}$  : Category Two Of Malament Hogarth Space Times

$T_{30}$  : Category Three Of Malament Hogarth Space Times

**Quantum Gravity Anthropic Computing**

$G_{32}$  : Category One Of Quantum Gravity (Total Gravity Exists)

$G_{33}$  : Category Two Of Quantum Gravity

$G_{34}$  : Category Three Of Quantum Gravity

$T_{32}$  : Category One Of Anthropic Computing

$T_{33}$  : Category Two Of Anthropic Computing

$T_{34}$  : Category Three Of Anthropic Computing

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$   
 $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}, (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$   
 $(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)},$   
 $(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$   
 $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$   
 $(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$   
 $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}, (a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

are Dissipation coefficients

### Governing Equations: System: Soap Bubble And Protein Folding

The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$  First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$  First detritions factor

### Governing Equations: System: Quantum Computing And Quantum Advice

The differential system of this model is now

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$  First augmentation factor

$-(b''_{16})^{(2)}((G_{19}), t) =$  First detritions factor

### Governing Equations: System: Quantum Adiabatic Algorithms And Quantum Mechanical Nonlinearities:

The differential system of this model is now

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22} \quad 18$$

$$+(a''_{20})^{(3)}(T_{21}, t) = \text{First augmentation factor}$$

$$-(b''_{20})^{(3)}(G_{23}, t) = \text{First detritions factor}$$

### Governing Equations: System: Hidden Variables And Relativistic Time Dilation

The differential system of this model is now

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)]T_{26} \quad 24$$

$$+(a''_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor}$$

$$-(b''_{24})^{(4)}((G_{27}), t) = \text{First detritions factor}$$

### Governing Equations: System: Analog Computing And Malament Hogarth Space Times

The differential system of this model is now

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)]T_{30} \quad 30$$

$$+(a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor}$$

$$-(b''_{28})^{(5)}((G_{31}), t) = \text{First detritions factor}$$

### Governing Equations: System: Quantum Gravity And Anthropic Computing

The differential system of this model is now

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)]T_{34} \quad 36$$

$$+(a''_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor}$$

$$-(b''_{32})^{(6)}((G_{35}), t) = \text{First detritions factor}$$

**System: Soap Bubble-Protein Folding –Quantum Computing-Quantum Advice-Quantum Adiabatic Algorithms –Quantum Mechanical Nonlinearities-Hidden Variables-Relativistic Time Dilation-Analog Computing-Malament Hogarth Space Times-Quantum Gravity-Anthropic Computing**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[ \begin{array}{c} (a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) + (a''_{16})^{(2,2)}(T_{17}, t) + (a''_{20})^{(3,3)}(T_{21}, t) \\ + (a''_{24})^{(4,4,4,4)}(T_{25}, t) + (a''_{28})^{(5,5,5,5)}(T_{29}, t) + (a''_{32})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{13} \quad 37$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[ \begin{array}{c} (a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t) + (a''_{17})^{(2,2)}(T_{17}, t) + (a''_{21})^{(3,3)}(T_{21}, t) \\ + (a''_{25})^{(4,4,4,4)}(T_{25}, t) + (a''_{29})^{(5,5,5,5)}(T_{29}, t) + (a''_{33})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{14} \quad 38$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[ \begin{array}{c} (a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t) + (a''_{18})^{(2,2)}(T_{17}, t) + (a''_{22})^{(3,3)}(T_{21}, t) \\ + (a''_{26})^{(4,4,4,4)}(T_{25}, t) + (a''_{30})^{(5,5,5,5)}(T_{29}, t) + (a''_{34})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{15} \quad 39$$

Where  $(a'_{13})^{(1)}(T_{14}, t)$ ,  $(a'_{14})^{(1)}(T_{14}, t)$ ,  $(a'_{15})^{(1)}(T_{14}, t)$  are first augmentation coefficients for category 1, 2 and 3

$(a'_{16})^{(2,2)}(T_{17}, t)$ ,  $(a'_{17})^{(2,2)}(T_{17}, t)$ ,  $(a'_{18})^{(2,2)}(T_{17}, t)$  are second augmentation coefficient

for category 1, 2 and 3

$(a'_{20})^{(3,3)}(T_{21}, t)$ ,  $(a'_{21})^{(3,3)}(T_{21}, t)$ ,  $(a'_{22})^{(3,3)}(T_{21}, t)$  are third augmentation coefficient

for category 1, 2 and 3

$(a'_{24})^{(4,4,4,4)}(T_{25}, t)$ ,  $(a'_{25})^{(4,4,4,4)}(T_{25}, t)$ ,  $(a'_{26})^{(4,4,4,4)}(T_{25}, t)$  are fourth augmentation

coefficient for category 1, 2 and 3

$(a'_{28})^{(5,5,5,5)}(T_{29}, t)$ ,  $(a'_{29})^{(5,5,5,5)}(T_{29}, t)$ ,  $(a'_{30})^{(5,5,5,5)}(T_{29}, t)$  are fifth augmentation

coefficient for category 1, 2 and 3

$(a'_{32})^{(6,6,6,6)}(T_{33}, t)$ ,  $(a'_{33})^{(6,6,6,6)}(T_{33}, t)$ ,  $(a'_{34})^{(6,6,6,6)}(T_{33}, t)$  are sixth augmentation

coefficient for category 1, 2 and 3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[ \begin{array}{c} (b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{16})^{(2,2)}(G_{19}, t) - (b''_{20})^{(3,3)}(G_{23}, t) \\ - (b''_{24})^{(4,4,4,4)}(G_{27}, t) - (b''_{28})^{(5,5,5,5)}(G_{31}, t) - (b''_{32})^{(6,6,6,6)}(G_{35}, t) \end{array} \right] T_{13} \quad 40$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[ \begin{array}{c} (b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t) - (b''_{17})^{(2,2)}(G_{19}, t) - (b''_{21})^{(3,3)}(G_{23}, t) \\ - (b''_{25})^{(4,4,4,4)}(G_{27}, t) - (b''_{29})^{(5,5,5,5)}(G_{31}, t) - (b''_{33})^{(6,6,6,6)}(G_{35}, t) \end{array} \right] T_{14} \quad 41$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[ \begin{array}{c} (b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t) - (b''_{18})^{(2,2)}(G_{19}, t) - (b''_{22})^{(3,3)}(G_{23}, t) \\ - (b''_{26})^{(4,4,4,4)}(G_{27}, t) - (b''_{30})^{(5,5,5,5)}(G_{31}, t) - (b''_{34})^{(6,6,6,6)}(G_{35}, t) \end{array} \right] T_{15} \quad 42$$

Where  $-(b''_{13})^{(1)}(G, t)$ ,  $-(b''_{14})^{(1)}(G, t)$ ,  $-(b''_{15})^{(1)}(G, t)$  are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2)}(G_{19}, t)$ ,  $-(b''_{17})^{(2,2)}(G_{19}, t)$ ,  $-(b''_{18})^{(2,2)}(G_{19}, t)$  are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3)}(G_{23}, t)$ ,  $-(b''_{21})^{(3,3)}(G_{23}, t)$ ,  $-(b''_{22})^{(3,3)}(G_{23}, t)$  are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4)}(G_{27}, t)$ ,  $-(b''_{25})^{(4,4,4,4)}(G_{27}, t)$ ,  $-(b''_{26})^{(4,4,4,4)}(G_{27}, t)$  are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5)}(G_{31}, t)$ ,  $-(b''_{29})^{(5,5,5,5)}(G_{31}, t)$ ,  $-(b''_{30})^{(5,5,5,5)}(G_{31}, t)$  are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6)}(G_{35}, t)$ ,  $-(b''_{33})^{(6,6,6,6)}(G_{35}, t)$ ,  $-(b''_{34})^{(6,6,6,6)}(G_{35}, t)$  are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[ \begin{array}{ccc} (a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) & + (a''_{13})^{(1,1)}(T_{14}, t) & + (a''_{20})^{(3,3)}(T_{21}, t) \\ + (a''_{24})^{(4,4,4,4)}(T_{25}, t) & + (a''_{28})^{(5,5,5,5)}(T_{29}, t) & + (a''_{32})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{16} \quad 43$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[ \begin{array}{ccc} (a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t) & + (a''_{14})^{(1,1)}(T_{14}, t) & + (a''_{21})^{(3,3)}(T_{21}, t) \\ + (a''_{25})^{(4,4,4,4)}(T_{25}, t) & + (a''_{29})^{(5,5,5,5)}(T_{29}, t) & + (a''_{33})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{17} \quad 44$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[ \begin{array}{ccc} (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t) & + (a''_{15})^{(1,1)}(T_{14}, t) & + (a''_{22})^{(3,3)}(T_{21}, t) \\ + (a''_{26})^{(4,4,4,4)}(T_{25}, t) & + (a''_{30})^{(5,5,5,5)}(T_{29}, t) & + (a''_{34})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{18} \quad 45$$

Where  $+(a''_{16})^{(2)}(T_{17}, t)$ ,  $+(a''_{17})^{(2)}(T_{17}, t)$ ,  $+(a''_{18})^{(2)}(T_{17}, t)$  are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1)}(T_{14}, t)$ ,  $+(a''_{14})^{(1,1)}(T_{14}, t)$ ,  $+(a''_{15})^{(1,1)}(T_{14}, t)$  are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3)}(T_{21}, t)$ ,  $+(a''_{21})^{(3,3)}(T_{21}, t)$ ,  $+(a''_{22})^{(3,3)}(T_{21}, t)$  are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4)}(T_{25}, t)$ ,  $+(a''_{25})^{(4,4,4,4)}(T_{25}, t)$ ,  $+(a''_{26})^{(4,4,4,4)}(T_{25}, t)$  are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5)}(T_{29}, t)$ ,  $+(a''_{29})^{(5,5,5,5)}(T_{29}, t)$ ,  $+(a''_{30})^{(5,5,5,5)}(T_{29}, t)$  are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6)}(T_{33}, t)$ ,  $+(a''_{33})^{(6,6,6,6)}(T_{33}, t)$ ,  $+(a''_{34})^{(6,6,6,6)}(T_{33}, t)$  are sixth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[ \begin{array}{ccc} (b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) & - (b''_{13})^{(1,1)}(G, t) & - (b''_{20})^{(3,3)}(G_{23}, t) \\ - (b''_{24})^{(4,4,4,4)}(G_{27}, t) & - (b''_{28})^{(5,5,5,5)}(G_{31}, t) & - (b''_{32})^{(6,6,6,6)}(G_{35}, t) \end{array} \right] T_{16} \quad 46$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[ \begin{array}{ccc} (b'_{17})^{(2)}[-(b''_{17})^{(2)}(G_{19}, t)] & -(b''_{14})^{(1,1)}(G, t) & -(b''_{21})^{(3,3,3)}(G_{23}, t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27}, t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31}, t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35}, t) \end{array} \right] T_{17} \quad 47$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[ \begin{array}{ccc} (b'_{18})^{(2)}[-(b''_{18})^{(2)}(G_{19}, t)] & -(b''_{15})^{(1,1)}(G, t) & -(b''_{22})^{(3,3,3)}(G_{23}, t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27}, t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31}, t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35}, t) \end{array} \right] T_{18} \quad 48$$

where  $[-(b''_{16})^{(2)}(G_{19}, t)]$ ,  $[-(b''_{17})^{(2)}(G_{19}, t)]$ ,  $[-(b''_{18})^{(2)}(G_{19}, t)]$  are first detrition coefficients for category 1, 2 and 3

$[-(b''_{13})^{(1,1)}(G, t)]$ ,  $[-(b''_{14})^{(1,1)}(G, t)]$ ,  $[-(b''_{15})^{(1,1)}(G, t)]$  are second detrition coefficients for category 1,2 and 3

$[-(b''_{20})^{(3,3,3)}(G_{23}, t)]$ ,  $[-(b''_{21})^{(3,3,3)}(G_{23}, t)]$ ,  $[-(b''_{22})^{(3,3,3)}(G_{23}, t)]$  are third detrition coefficients for category 1,2 and 3

$[-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)]$ ,  $[-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)]$ ,  $[-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)]$  are fourth detrition coefficients for category 1,2 and 3

$[-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)]$ ,  $[-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)]$ ,  $[-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)]$  are fifth detrition coefficients for category 1,2 and 3

$[-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)]$ ,  $[-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)]$ ,  $[-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)]$  are sixth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[ \begin{array}{ccc} (a'_{20})^{(3)}[+(a''_{20})^{(3)}(T_{21}, t)] & +(a''_{16})^{(2,2,2)}(T_{17}, t) & +(a''_{13})^{(1,1,1)}(T_{14}, t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25}, t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29}, t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33}, t) \end{array} \right] G_{20} \quad 49$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[ \begin{array}{ccc} (a'_{21})^{(3)}[+(a''_{21})^{(3)}(T_{21}, t)] & +(a''_{17})^{(2,2,2)}(T_{17}, t) & +(a''_{14})^{(1,1,1)}(T_{14}, t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25}, t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29}, t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33}, t) \end{array} \right] G_{21} \quad 50$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[ \begin{array}{ccc} (a'_{22})^{(3)}[+(a''_{22})^{(3)}(T_{21}, t)] & +(a''_{18})^{(2,2,2)}(T_{17}, t) & +(a''_{15})^{(1,1,1)}(T_{14}, t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25}, t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29}, t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33}, t) \end{array} \right] G_{22} \quad 51$$

$+(a''_{20})^{(3)}(T_{21}, t)$ ,  $+(a''_{21})^{(3)}(T_{21}, t)$ ,  $+(a''_{22})^{(3)}(T_{21}, t)$  are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17}, t)$ ,  $+(a''_{17})^{(2,2,2)}(T_{17}, t)$ ,  $+(a''_{18})^{(2,2,2)}(T_{17}, t)$  are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1)}(T_{14}, t)$ ,  $+(a''_{14})^{(1,1,1)}(T_{14}, t)$ ,  $+(a''_{15})^{(1,1,1)}(T_{14}, t)$  are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)$ ,  $+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)$ ,  $+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)$  are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)$ ,  $+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)$ ,  $+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)$  are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33}, t)}$  are sixth augmentation

coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[ \begin{array}{c} \boxed{(b'_{20})^{(3)}(G_{23}, t)} \quad \boxed{-(b''_{20})^{(3)}(G_{23}, t)} \quad \boxed{-(b'_{16})^{(2,2,2)}(G_{19}, t)} \quad \boxed{-(b'_{13})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27}, t)} \quad \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31}, t)} \quad \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{20} \quad 52$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[ \begin{array}{c} \boxed{(b'_{21})^{(3)}(G_{23}, t)} \quad \boxed{-(b''_{21})^{(3)}(G_{23}, t)} \quad \boxed{-(b'_{17})^{(2,2,2)}(G_{19}, t)} \quad \boxed{-(b'_{14})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27}, t)} \quad \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31}, t)} \quad \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{21} \quad 53$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[ \begin{array}{c} \boxed{(b'_{22})^{(3)}(G_{23}, t)} \quad \boxed{-(b''_{22})^{(3)}(G_{23}, t)} \quad \boxed{-(b'_{18})^{(2,2,2)}(G_{19}, t)} \quad \boxed{-(b'_{15})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27}, t)} \quad \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31}, t)} \quad \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{22} \quad 54$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b'_{16})^{(2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b'_{17})^{(2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b'_{18})^{(2,2,2)}(G_{19}, t)}$  are second detrition coefficients

for category 1, 2 and 3

$\boxed{-(b'_{13})^{(1,1,1)}(G, t)}$ ,  $\boxed{-(b'_{14})^{(1,1,1)}(G, t)}$ ,  $\boxed{-(b'_{15})^{(1,1,1)}(G, t)}$  are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27}, t)}$  are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31}, t)}$  are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35}, t)}$  are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[ \begin{array}{c} \boxed{(a'_{24})^{(4)}(T_{25}, t)} \quad \boxed{+(a''_{24})^{(4)}(T_{25}, t)} \quad \boxed{+(a'_{28})^{(5,5)}(T_{29}, t)} \quad \boxed{+(a'_{32})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a'_{13})^{(1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a'_{16})^{(2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{20})^{(3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{24} \quad 55$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[ \begin{array}{c} \boxed{(a'_{25})^{(4)}(T_{25}, t)} \quad \boxed{+(a''_{25})^{(4)}(T_{25}, t)} \quad \boxed{+(a'_{29})^{(5,5)}(T_{29}, t)} \quad \boxed{+(a'_{33})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a'_{14})^{(1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a'_{17})^{(2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{21})^{(3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{25} \quad 56$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[ \begin{array}{c} \boxed{(a'_{26})^{(4)}(T_{25}, t)} \quad \boxed{+(a''_{26})^{(4)}(T_{25}, t)} \quad \boxed{+(a'_{30})^{(5,5)}(T_{29}, t)} \quad \boxed{+(a'_{34})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a'_{15})^{(1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a'_{18})^{(2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{22})^{(3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{26} \quad 57$$

Where  $\boxed{(a'_{24})^{(4)}(T_{25}, t)}$ ,  $\boxed{(a'_{25})^{(4)}(T_{25}, t)}$ ,  $\boxed{(a'_{26})^{(4)}(T_{25}, t)}$  are first augmentation coefficients

for category 1, 2 and 3

$\boxed{+(a'_{28})^{(5,5)}(T_{29}, t)}$ ,  $\boxed{+(a'_{29})^{(5,5)}(T_{29}, t)}$ ,  $\boxed{+(a'_{30})^{(5,5)}(T_{29}, t)}$  are second augmentation coefficient

for category 1, 2 and 3

$\boxed{+(a'_{32})^{(6,6)}(T_{33}, t)}$ ,  $\boxed{+(a'_{33})^{(6,6)}(T_{33}, t)}$ ,  $\boxed{+(a'_{34})^{(6,6)}(T_{33}, t)}$  are third augmentation coefficient

for category 1, 2 and 3



$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)}$  are fourth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)}$  are fifth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21}, t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21}, t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21}, t)}$  are sixth augmentation coefficients for category 1, 2, and 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[ \begin{array}{ccc} \boxed{(b'_{24})^{(4)} \boxed{-(b''_{24})^{(4)}(G_{27}, t)} \boxed{-(b''_{28})^{(5,5)}(G_{31}, t)} \boxed{-(b''_{32})^{(6,6)}(G_{35}, t)}} & & \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G, t)} \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)} \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)} & & \end{array} \right] T_{24} \quad 58$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[ \begin{array}{ccc} \boxed{(b'_{25})^{(4)} \boxed{-(b''_{25})^{(4)}(G_{27}, t)} \boxed{-(b''_{29})^{(5,5)}(G_{31}, t)} \boxed{-(b''_{33})^{(6,6)}(G_{35}, t)}} & & \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G, t)} \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)} \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)} & & \end{array} \right] T_{25} \quad 59$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[ \begin{array}{ccc} \boxed{(b'_{26})^{(4)} \boxed{-(b''_{26})^{(4)}(G_{27}, t)} \boxed{-(b''_{30})^{(5,5)}(G_{31}, t)} \boxed{-(b''_{34})^{(6,6)}(G_{35}, t)}} & & \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G, t)} \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)} \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)} & & \end{array} \right] T_{26} \quad 60$$

Where  $\boxed{-(b''_{24})^{(4)}(G_{27}, t)}, \boxed{-(b''_{25})^{(4)}(G_{27}, t)}, \boxed{-(b''_{26})^{(4)}(G_{27}, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5)}(G_{31}, t)}, \boxed{-(b''_{29})^{(5,5)}(G_{31}, t)}, \boxed{-(b''_{30})^{(5,5)}(G_{31}, t)}$  are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6)}(G_{35}, t)}, \boxed{-(b''_{33})^{(6,6)}(G_{35}, t)}, \boxed{-(b''_{34})^{(6,6)}(G_{35}, t)}$  are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G, t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G, t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G, t)}$

are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)}$

are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)}$

are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[ \begin{array}{ccc} \boxed{(a'_{28})^{(5)} \boxed{+(a''_{28})^{(5)}(T_{29}, t)} \boxed{+(a''_{24})^{(4,4)}(T_{25}, t)} \boxed{+(a''_{32})^{(6,6)}(T_{33}, t)}} & & \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21}, t)} & & \end{array} \right] G_{28} \quad 61$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[ \begin{array}{ccc} \boxed{(a'_{29})^{(5)} \boxed{+(a''_{29})^{(5)}(T_{29}, t)} \boxed{+(a''_{25})^{(4,4)}(T_{25}, t)} \boxed{+(a''_{33})^{(6,6)}(T_{33}, t)}} & & \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21}, t)} & & \end{array} \right] G_{29} \quad 62$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[ \begin{array}{ccc} \boxed{(a'_{30})^{(5)} \boxed{+(a''_{30})^{(5)}(T_{29}, t)} \boxed{+(a''_{26})^{(4,4)}(T_{25}, t)} \boxed{+(a''_{34})^{(6,6)}(T_{33}, t)}} & & \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21}, t)} & & \end{array} \right] G_{30} \quad 63$$

Where  $\boxed{+(a''_{28})^{(5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5)}(T_{29}, t)}$  are first augmentation coefficients for category 1, 2 and 3

And  $\boxed{+(a''_{24})^{(4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{25})^{(4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{26})^{(4,4)}(T_{25}, t)}$  are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6,6)}(T_{33}, t)}$  are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)}$  are fourth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)}$  are fifth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)}$  are sixth augmentation coefficients for category 1, 2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[ \begin{array}{ccc} \boxed{(b'_{28})^{(5)}} & \boxed{-(b''_{28})^{(5)}(G_{31}, t)} & \boxed{-(b''_{24})^{(4,4)}(G_{27}, t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b'_{13})^{(1,1,1,1,1)}(G, t)} & \boxed{-(b'_{16})^{(2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b'_{20})^{(3,3,3,3,3)}(G_{23}, t)} & \end{array} \right] T_{28} \quad 64$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[ \begin{array}{ccc} \boxed{(b'_{29})^{(5)}} & \boxed{-(b''_{29})^{(5)}(G_{31}, t)} & \boxed{-(b''_{25})^{(4,4)}(G_{27}, t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b'_{14})^{(1,1,1,1,1)}(G, t)} & \boxed{-(b'_{17})^{(2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b'_{21})^{(3,3,3,3,3)}(G_{23}, t)} & \end{array} \right] T_{29} \quad 65$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[ \begin{array}{ccc} \boxed{(b'_{30})^{(5)}} & \boxed{-(b''_{30})^{(5)}(G_{31}, t)} & \boxed{-(b''_{26})^{(4,4)}(G_{27}, t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b'_{15})^{(1,1,1,1,1)}(G, t)} & \boxed{-(b'_{18})^{(2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b'_{22})^{(3,3,3,3,3)}(G_{23}, t)} & \end{array} \right] T_{30} \quad 66$$

where  $\boxed{-(b''_{28})^{(5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5)}(G_{31}, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4)}(G_{27}, t)}$  are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6,6)}(G_{35}, t)}$  are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b'_{13})^{(1,1,1,1,1)}(G, t)}$ ,  $\boxed{-(b'_{14})^{(1,1,1,1,1)}(G, t)}$ ,  $\boxed{-(b'_{15})^{(1,1,1,1,1)}(G, t)}$  are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b'_{16})^{(2,2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b'_{17})^{(2,2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b'_{18})^{(2,2,2,2,2)}(G_{19}, t)}$  are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b'_{20})^{(3,3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b'_{21})^{(3,3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b'_{22})^{(3,3,3,3,3)}(G_{23}, t)}$  are sixth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[ \begin{array}{ccc} \boxed{(a'_{32})^{(6)}} & \boxed{+(a''_{32})^{(6)}(T_{33}, t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29}, t)} & \boxed{+(a''_{24})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a'_{13})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a'_{16})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a'_{20})^{(3,3,3,3,3)}(T_{21}, t)} & \end{array} \right] G_{32} \quad 67$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \left[ \begin{array}{c} (a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t) + (a''_{29})^{(5,5,5)}(T_{29}, t) + (a''_{25})^{(4,4,4)}(T_{25}, t) \\ + (a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t) + (a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t) + (a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t) \end{array} \right] G_{33} \quad 68$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[ \begin{array}{c} (a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t) + (a''_{30})^{(5,5,5)}(T_{29}, t) + (a''_{26})^{(4,4,4)}(T_{25}, t) \\ + (a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t) + (a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t) + (a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t) \end{array} \right] G_{34} \quad 69$$

$+(a''_{32})^{(6)}(T_{33}, t), +(a''_{33})^{(6)}(T_{33}, t), +(a''_{34})^{(6)}(T_{33}, t)$  are first augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5)}(T_{29}, t), +(a''_{29})^{(5,5,5)}(T_{29}, t), +(a''_{30})^{(5,5,5)}(T_{29}, t)$  are second augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4)}(T_{25}, t), +(a''_{25})^{(4,4,4)}(T_{25}, t), +(a''_{26})^{(4,4,4)}(T_{25}, t)$  are third augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t), +(a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t), +(a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t)$  - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t), +(a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t), +(a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t)$  - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t), +(a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t), +(a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t)$  sixth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \left[ \begin{array}{c} (b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}, t) - (b''_{28})^{(5,5,5)}(G_{31}, t) - (b''_{24})^{(4,4,4)}(G_{27}, t) \\ - (b''_{13})^{(1,1,1,1,1,1)}(G, t) - (b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t) - (b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t) \end{array} \right] T_{32} \quad 70$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[ \begin{array}{c} (b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}, t) - (b''_{29})^{(5,5,5)}(G_{31}, t) - (b''_{25})^{(4,4,4)}(G_{27}, t) \\ - (b''_{14})^{(1,1,1,1,1,1)}(G, t) - (b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t) - (b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t) \end{array} \right] T_{33} \quad 71$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[ \begin{array}{c} (b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}, t) - (b''_{30})^{(5,5,5)}(G_{31}, t) - (b''_{26})^{(4,4,4)}(G_{27}, t) \\ - (b''_{15})^{(1,1,1,1,1,1)}(G, t) - (b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t) - (b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t) \end{array} \right] T_{34} \quad 72$$

$-(b''_{32})^{(6)}(G_{35}, t), -(b''_{33})^{(6)}(G_{35}, t), -(b''_{34})^{(6)}(G_{35}, t)$  are first detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5)}(G_{31}, t), -(b''_{29})^{(5,5,5)}(G_{31}, t), -(b''_{30})^{(5,5,5)}(G_{31}, t)$  are second detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4)}(G_{27}, t), -(b''_{25})^{(4,4,4)}(G_{27}, t), -(b''_{26})^{(4,4,4)}(G_{27}, t)$  are third detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G, t), -(b''_{14})^{(1,1,1,1,1,1)}(G, t), -(b''_{15})^{(1,1,1,1,1,1)}(G, t)$  are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t), -(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t), -(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)$  are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)}$  are sixth detrition coefficients for category 1, 2, and 3

Where we suppose

$$(A) \quad (a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i, j = 13, 14, 15 \quad 73$$

(B) The functions  $(a''_i)^{(1)}, (b''_i)^{(1)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(1)}, (r_i)^{(1)}$ :

$$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)} \\ (b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$(C) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)} \quad 74 \\ \lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$$

**Definition of**  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where  $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$  are positive constants and  $\boxed{i = 13, 14, 15}$

They satisfy Lipschitz condition:

$$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\hat{M}_{13})^{(1)}t} \\ |(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, t)| < (\hat{k}_{13})^{(1)} \|G - G'\| e^{-(\hat{M}_{13})^{(1)}t} \quad 75$$

With the Lipschitz condition, we place a restriction on the behavior of functions

$(a''_i)^{(1)}(T'_{14}, t)$  and  $(a''_i)^{(1)}(T_{14}, t)$ .  $(T'_{14}, t)$  and  $(T_{14}, t)$  are points belonging to the interval  $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$ . It is to be noted that  $(a''_i)^{(1)}(T_{14}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{13})^{(1)} = 1$  then the function  $(a''_i)^{(1)}(T_{14}, t)$ , the first augmentation coefficient Would be absolutely continuous.

**Definition of**  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 76

(D)  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ , are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of**  $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ :

(E) There exists two constants  $(\hat{P}_{13})^{(1)}$  and  $(\hat{Q}_{13})^{(1)}$  which together with  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$  and  $(\hat{B}_{13})^{(1)}$  and the constants  $(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15$ ,

satisfy the inequalities 77

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a'_i)^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1 \\ \frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b'_i)^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(F) \quad (a_i)^{(2)}, (a'_i)^{(2)}, (a''_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (b''_i)^{(2)} > 0, \quad i, j = 16, 17, 18$$

(G) The functions  $(a''_i)^{(2)}, (b''_i)^{(2)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(2)}, (r_i)^{(2)}$ :

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$$\begin{aligned}(a_i'')^{(2)}(T_{17}, t) &\leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \\ (b_i'')^{(2)}(G_{19}, t) &\leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \\ (H) \quad \lim_{T_2 \rightarrow \infty} (a_i'')^{(2)}(T_{17}, t) &= (p_i)^{(2)} \\ \lim_{G \rightarrow \infty} (b_i'')^{(2)}(G_{19}, t) &= (r_i)^{(2)}\end{aligned}$$

**Definition of**  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ :

79

Where  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$  are positive constants and  $i = 16, 17, 18$

They satisfy Lipschitz condition:

$$\begin{aligned}|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| &\leq (\hat{k}_{16})^{(2)} |T_{17}' - T_{17}| e^{-(\hat{M}_{16})^{(2)} t} \\ |(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| &< (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t}\end{aligned}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(2)}(T_{17}', t)$  and  $(a_i'')^{(2)}(T_{17}, t)$ .  $(T_{17}', t)$  and  $(T_{17}, t)$  are points belonging to the interval  $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that  $(a_i'')^{(2)}(T_{17}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{16})^{(2)} = 1$  then the function  $(a_i'')^{(2)}(T_{17}, t)$ , the SECOND augmentation coefficient would be absolutely continuous.

**Definition of**  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

80

(I)  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ , are positive constants

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of**  $(\hat{P}_{16})^{(2)}, (\hat{Q}_{16})^{(2)}$ :

81

There exists two constants  $(\hat{P}_{16})^{(2)}$  and  $(\hat{Q}_{16})^{(2)}$  which together with  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$  and  $(\hat{B}_{16})^{(2)}$  and the constants  $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18$ , satisfy the inequalities

$$\begin{aligned}\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] &< 1 \\ \frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] &< 1\end{aligned}$$

Where we suppose

(J)  $(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, i, j = 20, 21, 22$

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The functions  $(a_i'')^{(3)}, (b_i'')^{(3)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(3)}, (r_i)^{(3)}$ :

$$\begin{aligned}(a_i'')^{(3)}(T_{21}, t) &\leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)} \\ (b_i'')^{(3)}(G_{23}, t) &\leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)} \\ \lim_{T_2 \rightarrow \infty} (a_i'')^{(3)}(T_{21}, t) &= (p_i)^{(3)} \\ \lim_{G \rightarrow \infty} (b_i'')^{(3)}(G_{23}, t) &= (r_i)^{(3)}\end{aligned}$$

**Definition of**  $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

83

Where  $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}$  are positive constants and  $i = 20, 21, 22$

They satisfy Lipschitz condition:

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)}t} \quad 84$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} \|G_{23} - G_{23}'\| e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(3)}(T_{21}', t)$  and  $(a_i'')^{(3)}(T_{21}, t)$ .  $(T_{21}', t)$  and  $(T_{21}, t)$  are points belonging to the interval  $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$ . It is to be noted that  $(a_i'')^{(3)}(T_{21}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{20})^{(3)} = 1$  then the function  $(a_i'')^{(3)}(T_{21}, t)$ , the THIRD augmentation coefficient would be absolutely continuous.

**Definition of**  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$  : 85

(K)  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ , are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants  $(\hat{P}_{20})^{(3)}$  and  $(\hat{Q}_{20})^{(3)}$  which together with  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$  and  $(\hat{B}_{20})^{(3)}$  and the constants 86

$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22$ ,

satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

(L)  $(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, i, j = 24, 25, 26$  87

(M) The functions  $(a_i'')^{(4)}, (b_i'')^{(4)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(4)}, (r_i)^{(4)}$ : 88

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

(N)  $\lim_{T_2 \rightarrow \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$  89

$$\lim_{G \rightarrow \infty} (b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of**  $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$  :

Where  $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}$  are positive constants and  $i = 24, 25, 26$

They satisfy Lipschitz condition: 90

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)} \|(G_{27}) - (G_{27}')\| e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(4)}(T_{25}', t)$  and  $(a_i'')^{(4)}(T_{25}, t)$ .  $(T_{25}', t)$  and  $(T_{25}, t)$  are points belonging to the interval  $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$ . It is to be noted that  $(a_i'')^{(4)}(T_{25}, t)$  is uniformly continuous. In the eventuality of the fact, that if

$(\hat{M}_{24})^{(4)} = 1$  then the function  $(a_i'')^{(4)}(T_{25}, t)$ , the FOURTH **augmentation coefficient**, would be absolutely continuous.

**Definition of**  $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$  :

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(O)  $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ , are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of**  $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$  :

92

(P) There exists two constants  $(\hat{P}_{24})^{(4)}$  and  $(\hat{Q}_{24})^{(4)}$  which together with

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}$  and  $(\hat{B}_{24})^{(4)}$  and the constants

$(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26$ ,

satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

(Q)  $(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, i, j = 28, 29, 30$

93

(R) The functions  $(a_i'')^{(5)}, (b_i'')^{(5)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(5)}, (r_i)^{(5)}$ :

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

(S)  $\lim_{T_2 \rightarrow \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$

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$$\lim_{G \rightarrow \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of**  $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$  :

Where  $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$  are positive constants and  $i = 28, 29, 30$

They satisfy Lipschitz condition:

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$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)} |T_{29}' - T_{29}| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31})' - (G_{31})|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(5)}(T_{29}', t)$

and  $(a_i'')^{(5)}(T_{29}, t)$ .  $(T_{29}', t)$  and  $(T_{29}, t)$  are points belonging to the interval  $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$ . It

is to be noted that  $(a_i'')^{(5)}(T_{29}, t)$  is uniformly continuous. In the eventuality of the fact, that if

$(\hat{M}_{28})^{(5)} = 1$  then the function  $(a_i'')^{(5)}(T_{29}, t)$ , the FIFTH **augmentation coefficient** would be absolutely continuous.

**Definition of**  $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$  :

96

(T)  $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ , are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of**  $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$  :

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(U) There exists two constants  $(\hat{P}_{28})^{(5)}$  and  $(\hat{Q}_{28})^{(5)}$  which together with  $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}$  and  $(\hat{B}_{28})^{(5)}$  and the constants  $(a_i)^{(5)}, (a'_i)^{(5)}, (b_i)^{(5)}, (b'_i)^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(a_i)^{(5)} + (a'_i)^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(b_i)^{(5)} + (b'_i)^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a'_i)^{(6)}, (a''_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (b''_i)^{(6)} > 0, \quad i, j = 32, 33, 34 \quad 98$$

(V) The functions  $(a''_i)^{(6)}, (b''_i)^{(6)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(6)}, (r_i)^{(6)}$ :

$$(a''_i)^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b''_i)^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b'_i)^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$(W) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(6)}(T_{33}, t) = (p_i)^{(6)} \quad 99$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of**  $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where  $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}$  are positive constants and  $i = 32, 33, 34$

They satisfy Lipschitz condition: 100

$$|(a''_i)^{(6)}(T'_{33}, t) - (a''_i)^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T'_{33}| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b''_i)^{(6)}((G_{35})', t) - (b''_i)^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} |(G_{35}) - (G_{35})'| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(6)}(T'_{33}, t)$  and  $(a''_i)^{(6)}(T_{33}, t)$ .  $(T'_{33}, t)$  and  $(T_{33}, t)$  are points belonging to the interval  $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$ . It is to be noted that  $(a''_i)^{(6)}(T_{33}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{32})^{(6)} = 1$  then the function  $(a''_i)^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

**Definition of**  $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 101

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ , are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of**  $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 102

There exists two constants  $(\hat{P}_{32})^{(6)}$  and  $(\hat{Q}_{32})^{(6)}$  which together with

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)}$  and  $(\hat{B}_{32})^{(6)}$  and the constants

$(a_i)^{(6)}, (a'_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32, 33, 34$ ,

satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(a_i)^{(6)} + (a'_i)^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(b_i)^{(6)} + (b'_i)^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$



**Theorem 1:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

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**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 1:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

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**Definition of**  $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

**Theorem 1:** if the conditions IN THE FOREGOING, NAMELY FIRST FIVE CONDITIONS are fulfilled, there exists a solution satisfying the conditions

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

if the conditions SECOND FIVE CONDITIONS above are fulfilled, there exists a solution satisfying the conditions

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**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

if the conditions THIRD MODULE OF FIVE CONDITIONS above are fulfilled, there exists a solution satisfying the conditions

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**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

if the conditions FOURTH MODULE OF FIVE CONDITIONS CONCOMITANT TO A-E above are fulfilled, there exists a solution satisfying the conditions

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**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator  $\mathcal{A}^{(1)}$  defined on the space of sextuples of continuous functions

$G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}$$

By 108

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)} \quad 109$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)} \quad 110$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)} \quad 111$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)} \quad 112$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)} \quad 113$$

Where  $s_{(13)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(2)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  114

which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 115

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16}(T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)} \quad 116$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)} \quad 117$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)} \quad 118$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)} \quad 119$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)} \quad 120$$

Where  $s_{(16)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(3)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  121

which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 122

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20}(T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)} \quad 123$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)} \quad 124$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)} \quad 125$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)} \quad 126$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)} \quad 127$$

Where  $s_{(20)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(4)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{24})^{(4)}, T_i^0 \leq (\hat{Q}_{24})^{(4)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 129

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24}(T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)} \quad 130$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[ (a_{25})^{(4)} G_{24}(s_{(24)}) - \left( (a'_{25})^{(4)} + a''_{25}(T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)} \quad 131$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[ (a_{26})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{26})^{(4)} + a''_{26}(T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)} \quad 132$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[ (b_{24})^{(4)} T_{25}(s_{(24)}) - \left( (b'_{24})^{(4)} - (b''_{24})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)} \quad 133$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[ (b_{25})^{(4)} T_{24}(s_{(24)}) - \left( (b'_{25})^{(4)} - (b''_{25})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)} \quad 134$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[ (b_{26})^{(4)} T_{25}(s_{(24)}) - \left( (b'_{26})^{(4)} - (b''_{26})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)} \quad 135$$

Where  $s_{(24)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(5)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{28})^{(5)}, T_i^0 \leq (\hat{Q}_{28})^{(5)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 136

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{28})^{(5)} + a''_{28}(T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)} \quad 137$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{28}(s_{(28)}) - \left( (a'_{29})^{(5)} + a''_{29}(T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)} \quad 138$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{30})^{(5)} + a''_{30}(T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)} \quad 139$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{28})^{(5)} - (b''_{28})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)} \quad 140$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})^{(5)} T_{28}(s_{(28)}) - \left( (b'_{29})^{(5)} - (b''_{29})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)} \quad 141$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{30})^{(5)} - (b''_{30})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)} \quad 142$$

Where  $s_{(28)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(6)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{32})^{(6)}, T_i^0 \leq (\hat{Q}_{32})^{(6)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

By

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$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32}(T_{33}(s_{(32)}), s_{(32)})) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + a''_{33}(T_{33}(s_{(32)}), s_{(32)})) G_{33}(s_{(32)}) \right] ds_{(32)} \quad 144$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + a''_{34}(T_{33}(s_{(32)}), s_{(32)})) G_{34}(s_{(32)}) \right] ds_{(32)} \quad 145$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)}(G(s_{(32)}), s_{(32)})) T_{32}(s_{(32)}) \right] ds_{(32)} \quad 146$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)}(G(s_{(32)}), s_{(32)})) T_{33}(s_{(32)}) \right] ds_{(32)} \quad 147$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)}(G(s_{(32)}), s_{(32)})) T_{34}(s_{(32)}) \right] ds_{(32)} \quad 148$$

Where  $s_{(32)}$  is the integrand that is integrated over an interval  $(0, t)$

(a) The operator  $\mathcal{A}^{(1)}$  maps the space of functions satisfying the system into itself. Indeed it is obvious that 149

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} \left( G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left( 1 + (a_{13})^{(1)} t \right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left( e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that

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$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ \left( (\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}} + (\hat{P}_{13})^{(1)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem

Analogous inequalities hold also for  $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(b) The operator  $\mathcal{A}^{(2)}$  maps the space of functions into itself. Indeed it is obvious that 151

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} \left( G_{17}^0 + (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \left( 1 + (a_{16})^{(2)} t \right) G_{17}^0 +$$

$$\frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left( e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[ \left( (\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for  $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

(a) The operator  $\mathcal{A}^{(3)}$  maps the space of functions into itself. Indeed it is obvious that 152

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} \left( G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}} \right) \right] ds_{(20)} =$$

$$\left( 1 + (a_{20})^{(3)} t \right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left( e^{(\hat{M}_{20})^{(3)} t} - 1 \right)$$

From which it follows that

$$(G_{20}(t) - G_{20}^0)e^{-(\bar{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\bar{M}_{20})^{(3)}} \left[ ((\hat{P}_{20})^{(3)} + G_{21}^0)e^{-\left(\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for  $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

(b) The operator  $\mathcal{A}^{(4)}$  maps the space of functions into itself. Indeed it is obvious that 153

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} \left( G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)}s_{(24)}} \right) \right] ds_{(24)} = \\ (1 + (a_{24})^{(4)}t)G_{25}^0 + \frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\bar{M}_{24})^{(4)}} \left( e^{(\bar{M}_{24})^{(4)}t} - 1 \right)$$

From which it follows that 154

$$(G_{24}(t) - G_{24}^0)e^{-(\bar{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\bar{M}_{24})^{(4)}} \left[ ((\hat{P}_{24})^{(4)} + G_{25}^0)e^{-\left(\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem

(c) The operator  $\mathcal{A}^{(5)}$  maps the space of functions into itself. Indeed it is obvious that 155

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left( G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\bar{M}_{28})^{(5)}s_{(28)}} \right) \right] ds_{(28)} = \\ (1 + (a_{28})^{(5)}t)G_{29}^0 + \frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\bar{M}_{28})^{(5)}} \left( e^{(\bar{M}_{28})^{(5)}t} - 1 \right)$$

From which it follows that 156

$$(G_{28}(t) - G_{28}^0)e^{-(\bar{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\bar{M}_{28})^{(5)}} \left[ ((\hat{P}_{28})^{(5)} + G_{29}^0)e^{-\left(\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}\right)} + (\hat{P}_{28})^{(5)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem

(d) The operator  $\mathcal{A}^{(6)}$  maps the space of functions satisfying the system into itself. Indeed it is obvious that 157

$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}s_{(32)}} \right) \right] ds_{(32)} = \\ (1 + (a_{32})^{(6)}t)G_{33}^0 + \frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\bar{M}_{32})^{(6)}} \left( e^{(\bar{M}_{32})^{(6)}t} - 1 \right)$$

From which it follows that 158

$$(G_{32}(t) - G_{32}^0)e^{-(\bar{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\bar{M}_{32})^{(6)}} \left[ ((\hat{P}_{32})^{(6)} + G_{33}^0)e^{-\left(\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem

Analogous inequalities hold also for  $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

It is now sufficient to take  $\frac{(a_i)^{(1)}}{(\bar{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\bar{M}_{13})^{(1)}} < 1$  and to choose 159

$(\hat{P}_{13})^{(1)}$  and  $(\hat{Q}_{13})^{(1)}$  large to have

$$\frac{(a_i)^{(1)}}{(\bar{M}_{13})^{(1)}} \left[ ((\hat{P}_{13})^{(1)} + ((\hat{P}_{13})^{(1)} + G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}) \right] \leq (\hat{P}_{13})^{(1)} \quad 160$$

$$\frac{(b_i)^{(1)}}{(\bar{M}_{13})^{(1)}} \left[ ((\hat{Q}_{13})^{(1)} + T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)} \quad 161$$

In order that the operator  $\mathcal{A}^{(1)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself

The operator  $\mathcal{A}^{(1)}$  is a contraction with respect to the metric

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$$d\left((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)})\right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{13})^{(1)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{13})^{(1)}t} \right\}$$

Indeed if we denote

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**Definition of  $\tilde{G}, \tilde{T}$ :**  $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\begin{aligned} |\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} ds_{(13)} + \\ &\int_0^t \{ (a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &G_{13}^{(2)} | (a'_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) - (a'_{13})^{(1)} (T_{14}^{(2)}, s_{(13)}) | e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)} \end{aligned}$$

Where  $s_{(13)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{13})^{(1)}t} &\leq \\ \frac{1}{(\widehat{M}_{13})^{(1)}} &((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}) d\left((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)})\right) \end{aligned}$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a'_{13})^{(1)}$  and  $(b'_{13})^{(1)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$$(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \text{ and } (\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \text{ respectively of } \mathbb{R}_+.$$

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact

then it suffices to consider that  $(a'_i)^{(1)}$  and  $(b'_i)^{(1)}$ ,  $i = 13, 14, 15$  depend only on  $T_{14}$  and

respectively on  $G$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$

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From GOVERNING EQUATIONS it results

$$G_i(t) \geq G_i^0 e^{\left[- \int_0^t \{ (a'_i)^{(1)} - (a''_i)^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \} ds_{(13)} \right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(1)}t} > 0 \text{ for } t > 0$$

**Definition of  $((\widehat{M}_{13})^{(1)})_1, ((\widehat{M}_{13})^{(1)})_2$  and  $((\widehat{M}_{13})^{(1)})_3$ :**

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**Remark 3:** if  $G_{13}$  is bounded, the same property have also  $G_{14}$  and  $G_{15}$ . indeed if

$$G_{13} < ((\widehat{M}_{13})^{(1)}) \text{ it follows } \frac{dG_{14}}{dt} \leq ((\widehat{M}_{13})^{(1)})_1 - (a'_{14})^{(1)} G_{14} \text{ and by integrating}$$

$$G_{14} \leq ((\widehat{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)} ((\widehat{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)} ((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If  $G_{14}$  or  $G_{15}$  is bounded, the same property follows for  $G_{13}$ ,  $G_{15}$  and  $G_{13}$ ,  $G_{14}$  respectively.

**Remark 4:** If  $G_{13}$  is bounded, from below, the same property holds for  $G_{14}$  and  $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{14}$  is bounded from below.

**Remark 5:** If  $T_{13}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$  then  $T_{14} \rightarrow \infty$ . 166

**Definition of**  $(m)^{(1)}$  and  $\varepsilon_1$  :

Indeed let  $t_1$  be so that for  $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then  $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$  which leads to 167

$$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}$$

$$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_1} \text{ By taking now } \varepsilon_1 \text{ sufficiently small one sees that } T_{14} \text{ is}$$

unbounded. The same property holds for  $T_{15}$  if  $\lim_{t \rightarrow \infty} (b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take  $\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\bar{M}_{16})^{(2)}} < 1$  and to choose 168

$(\hat{P}_{16})^{(2)}$  and  $(\hat{Q}_{16})^{(2)}$  large to have

$$\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[ (\hat{P}_{16})^{(2)} + ((\hat{P}_{16})^{(2)} + G_j^0) e^{-\left( \frac{(\hat{P}_{16})^{(2)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{16})^{(2)} \quad 169$$

$$\frac{(b_i)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[ ((\hat{Q}_{16})^{(2)} + T_j^0) e^{-\left( \frac{(\hat{Q}_{16})^{(2)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{16})^{(2)} \right] \leq (\hat{Q}_{16})^{(2)} \quad 170$$

In order that the operator  $\mathcal{A}^{(2)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself

The operator  $\mathcal{A}^{(2)}$  is a contraction with respect to the metric 171

$$d \left( ((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t} \right\}$$

Indeed if we denote

$$\text{Definition of } \widetilde{G}_{19}, \widetilde{T}_{19} : (\widetilde{G}_{19}, \widetilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$$

It results

$$\begin{aligned} |\widetilde{G}_{16}^{(1)} - \widetilde{G}_{16}^{(2)}| &\leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} ds_{(16)} + \\ &\int_0^t \{ (a_{16}')^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{-(\bar{M}_{16})^{(2)}s_{(16)}} + \\ &(a_{16}'')^{(2)} (T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} + \\ &G_{16}^{(2)} |(a_{16}'')^{(2)} (T_{17}^{(1)}, s_{(16)}) - (a_{16}'')^{(2)} (T_{17}^{(2)}, s_{(16)})| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} \} ds_{(16)} \end{aligned}$$

Where  $s_{(16)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$|(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\bar{M}_{16})^{(2)}t} \leq$$

$$\frac{1}{(\widehat{M}_{16})^{(2)}}((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)})d\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a'_{16})^{(2)}$  and  $(b'_{16})^{(2)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$$(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t} \text{ and } (\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t} \text{ respectively of } \mathbb{R}_+.$$

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(2)}$  and  $(b''_i)^{(2)}$ ,  $i = 16, 17, 18$  depend only on  $T_{17}$  and respectively on  $(G_{19})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  172

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(2)} - (a''_i)^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(2)}t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{16})^{(2)})_1, ((\widehat{M}_{16})^{(2)})_2$  and  $((\widehat{M}_{16})^{(2)})_3$  : 173

**Remark 3:** if  $G_{16}$  is bounded, the same property have also  $G_{17}$  and  $G_{18}$ . indeed if

$$G_{16} < ((\widehat{M}_{16})^{(2)}) \text{ it follows } \frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a'_{17})^{(2)}G_{17} \text{ and by integrating}$$

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)}((\widehat{M}_{16})^{(2)})_1 / (a'_{17})^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)}((\widehat{M}_{16})^{(2)})_2 / (a'_{18})^{(2)}$$

If  $G_{17}$  or  $G_{18}$  is bounded, the same property follows for  $G_{16}$ ,  $G_{18}$  and  $G_{16}$ ,  $G_{17}$  respectively.

**Remark 4:** If  $G_{16}$  is bounded, from below, the same property holds for  $G_{17}$  and  $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{17}$  is bounded from below.

**Remark 5:** If  $T_{16}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b'_i)^{(2)}((G_{19})(t), t)) = (b'_{17})^{(2)}$  then  $T_{17} \rightarrow \infty$ .

**Definition of**  $(m)^{(2)}$  and  $\varepsilon_2$  :

Indeed let  $t_2$  be so that for  $t > t_2$

$$(b_{17})^{(2)} - (b'_i)^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then  $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$  which leads to

$$T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_2 t} = \frac{1}{2} \text{ it results}$$

$$T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_2} \text{ By taking now } \varepsilon_2 \text{ sufficiently small one sees that } T_{17} \text{ is}$$

unbounded. The same property holds for  $T_{18}$  if  $\lim_{t \rightarrow \infty} (b'_{18})^{(2)}((G_{19})(t), t) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of

SOLUTIONAL EQUATIONS OF THE HOLISTIC GOVERNING EQUATIONS

It is now sufficient to take  $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$  and to choose



$(\hat{P}_{20})^{(3)}$  and  $(\hat{Q}_{20})^{(3)}$  large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[ (\hat{P}_{20})^{(3)} + ((\hat{P}_{20})^{(3)} + G_j^0) e^{-\left(\frac{(\hat{P}_{20})^{(3)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{20})^{(3)} \quad 174$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[ ((\hat{Q}_{20})^{(3)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{20})^{(3)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)} \right] \leq (\hat{Q}_{20})^{(3)}$$

In order that the operator  $\mathcal{A}^{(3)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself

The operator  $\mathcal{A}^{(3)}$  is a contraction with respect to the metric 175

$$d\left((G_{23})^{(1)}, (T_{23})^{(1)}, (G_{23})^{(2)}, (T_{23})^{(2)}\right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t} \right\}$$

Indeed if we denote

**Definition of**  $\widetilde{G_{23}}, \widetilde{T_{23}} : (\widetilde{G_{23}}, \widetilde{T_{23}}) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 176

$$|\tilde{G}_{20}^{(1)} - \tilde{G}_{20}^{(2)}| \leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{-(\bar{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)} |(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) - (a''_{20})^{(3)} (T_{21}^{(2)}, s_{(20)})| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where  $s_{(20)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$|G^{(1)} - G^{(2)}| e^{-(\bar{M}_{20})^{(3)}t} \leq$$

$$\frac{1}{(\bar{M}_{20})^{(3)}} ((a_{20})^{(3)} + (a'_{20})^{(3)} + (\bar{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}) d\left((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}\right)$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a''_{20})^{(3)}$  and  $(b''_{20})^{(3)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)}t}$  and  $(\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)}t}$  respectively of  $\mathbb{R}_+$ .

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact

then it suffices to consider that  $(a''_i)^{(3)}$  and  $(b''_i)^{(3)}$ ,  $i = 20, 21, 22$  depend only on  $T_{21}$  and respectively on  $(G_{23})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$

From THE SOLUTIONS TO THE GOVERNING EQUATIONS AND THE CONCATENATED EQUATIONS it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(3)} - (a''_i)^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i^{(3)})t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{20})^{(3)})_1, ((\widehat{M}_{20})^{(3)})_2$  and  $((\widehat{M}_{20})^{(3)})_3$  :

**Remark 3:** if  $G_{20}$  is bounded, the same property have also  $G_{21}$  and  $G_{22}$  . indeed if

$$G_{20} < ((\widehat{M}_{20})^{(3)}) \text{ it follows } \frac{dG_{21}}{dt} \leq ((\widehat{M}_{20})^{(3)})_1 - (a'_{21})^{(3)} G_{21} \text{ and by integrating}$$

$$G_{21} \leq ((\widehat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)}((\widehat{M}_{20})^{(3)})_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq ((\widehat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)}((\widehat{M}_{20})^{(3)})_2 / (a'_{22})^{(3)}$$

If  $G_{21}$  or  $G_{22}$  is bounded, the same property follows for  $G_{20}$  ,  $G_{22}$  and  $G_{20}$  ,  $G_{21}$  respectively.

**Remark 4:** If  $G_{20}$  is bounded, from below, the same property holds for  $G_{21}$  and  $G_{22}$  . The proof is analogous with the preceding one. An analogous property is true if  $G_{21}$  is bounded from below.

**Remark 5:** If  $T_{20}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(3)}((G_{23})(t), t)) = (b'_{21})^{(3)}$  then  $T_{21} \rightarrow \infty$ .

**Definition of**  $(m)^{(3)}$  and  $\varepsilon_3$  :

Indeed let  $t_3$  be so that for  $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then  $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$  which leads to

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$$T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_3 t} = \frac{1}{2} \text{ it results}$$

$T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{2} \right), t = \log \frac{2}{\varepsilon_3}$  By taking now  $\varepsilon_3$  sufficiently small one sees that  $T_{21}$  is unbounded. The same property holds for  $T_{22}$  if  $\lim_{t \rightarrow \infty} (b_{22}'')^{(3)}((G_{23})(t), t) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions OF THE CONCATENATED GOVENING EQUATIONS

It is now sufficient to take  $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$  and to choose

$(\widehat{P}_{24})^{(4)}$  and  $(\widehat{Q}_{24})^{(4)}$  large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \left[ (\widehat{P}_{24})^{(4)} + ((\widehat{P}_{24})^{(4)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{24})^{(4)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{24})^{(4)} \quad 178$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \left[ ((\widehat{Q}_{24})^{(4)} + T_j^0) e^{-\left( \frac{(\widehat{Q}_{24})^{(4)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{24})^{(4)} \right] \leq (\widehat{Q}_{24})^{(4)} \quad 179$$

In order that the operator  $\mathcal{A}^{(4)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself

The operator  $\mathcal{A}^{(4)}$  is a contraction with respect to the metric

$$d \left( ((G_{27})^{(1)}, (T_{27})^{(1)}), ((G_{27})^{(2)}, (T_{27})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{24})^{(4)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{24})^{(4)}t} \}$$

Indeed if we denote

**Definition of**  $(\widetilde{G_{27}}), (\widetilde{T_{27}}) : (\widetilde{G_{27}}, \widetilde{T_{27}}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\begin{aligned} |\tilde{G}_{24}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{24})^{(4)} |G_{25}^{(1)} - G_{25}^{(2)}| e^{-(\widetilde{M}_{24})^{(4)} s_{(24)}} e^{(\widetilde{M}_{24})^{(4)} s_{(24)}} ds_{(24)} + \\ &\int_0^t \{(a'_{24})^{(4)} |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\widetilde{M}_{24})^{(4)} s_{(24)}} e^{-(\widetilde{M}_{24})^{(4)} s_{(24)}} + \\ &(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\widetilde{M}_{24})^{(4)} s_{(24)}} e^{(\widetilde{M}_{24})^{(4)} s_{(24)}} + \\ &G_{24}^{(2)} |(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) - (a''_{24})^{(4)} (T_{25}^{(2)}, s_{(24)})| e^{-(\widetilde{M}_{24})^{(4)} s_{(24)}} e^{(\widetilde{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)} \end{aligned}$$

Where  $s_{(24)}$  represents integrand that is integrated over the interval  $[0, t]$

it follows

$$\begin{aligned} |(G_{27})^{(1)} - (G_{27})^{(2)}| e^{-(\widetilde{M}_{24})^{(4)} t} &\leq \\ \frac{1}{(\widetilde{M}_{24})^{(4)}} &((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}) d \left( ((G_{27})^{(1)}, (T_{27})^{(1)}; (G_{27})^{(2)}, (T_{27})^{(2)}) \right) \end{aligned}$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis The result follows

**Remark 1:** The fact that we supposed  $(a''_{24})^{(4)}$  and  $(b''_{24})^{(4)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{24})^{(4)} e^{(\widetilde{M}_{24})^{(4)} t}$  and  $(\widehat{Q}_{24})^{(4)} e^{(\widetilde{M}_{24})^{(4)} t}$  respectively of  $\mathbb{R}_+$ .

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(4)}$  and  $(b''_i)^{(4)}$ ,  $i = 24, 25, 26$  depend only on  $T_{25}$  and respectively on  $(G_{27})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$

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it results

$$\begin{aligned} G_i(t) &\geq G_i^0 e^{[-\int_0^t \{(a'_i)^{(4)} - (a''_i)^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}]} \geq 0 \\ T_i(t) &\geq T_i^0 e^{-(b'_i)^{(4)} t} > 0 \text{ for } t > 0 \end{aligned}$$

**Definition of**  $((\widetilde{M}_{24})^{(4)})_1, ((\widetilde{M}_{24})^{(4)})_2$  and  $((\widetilde{M}_{24})^{(4)})_3 :$

**Remark 3:** if  $G_{24}$  is bounded, the same property have also  $G_{25}$  and  $G_{26}$ . indeed if

$$\begin{aligned} G_{24} < ((\widetilde{M}_{24})^{(4)})_1 &\text{ it follows } \frac{dG_{25}}{dt} \leq ((\widetilde{M}_{24})^{(4)})_1 - (a'_{25})^{(4)} G_{25} \text{ and by integrating} \\ G_{25} \leq ((\widetilde{M}_{24})^{(4)})_2 &= G_{25}^0 + 2(a_{25})^{(4)} ((\widetilde{M}_{24})^{(4)})_1 / (a'_{25})^{(4)} \end{aligned}$$

In the same way, one can obtain

$$G_{26} \leq ((\widetilde{M}_{24})^{(4)})_3 = G_{26}^0 + 2(a_{26})^{(4)} ((\widetilde{M}_{24})^{(4)})_2 / (a'_{26})^{(4)}$$

If  $G_{25}$  or  $G_{26}$  is bounded, the same property follows for  $G_{24}$ ,  $G_{26}$  and  $G_{24}$ ,  $G_{25}$  respectively.

**Remark 4:** If  $G_{24}$  is bounded, from below, the same property holds for  $G_{25}$  and  $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{25}$  is bounded from below.

**Remark 5:** If  $T_{24}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b'_i)^{(4)}((G_{27})(t), t)) = (b'_{25})^{(4)}$  then  $T_{25} \rightarrow \infty$ .

**Definition of**  $(m)^{(4)}$  and  $\varepsilon_4 :$

Indeed let  $t_4$  be so that for  $t > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then  $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$  which leads to

$$T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4} \right) (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_4 t} = \frac{1}{2} \text{ it results}$$

$T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_4}$  By taking now  $\varepsilon_4$  sufficiently small one sees that  $T_{25}$  is unbounded. The same property holds for  $T_{26}$  if  $\lim_{t \rightarrow \infty} (b_{26}'')^{(4)}((G_{27})(t), t) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of THE CONCATENATED EQUATIONS

Analogous inequalities hold also for  $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take  $\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} < 1$  and to choose

$(\hat{P}_{28})^{(5)}$  and  $(\hat{Q}_{28})^{(5)}$  large to have

$$\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}} \left[ (\hat{P}_{28})^{(5)} + ((\hat{P}_{28})^{(5)} + G_j^0) e^{-\left( \frac{(\hat{P}_{28})^{(5)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{28})^{(5)}$$

$$\frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} \left[ ((\hat{Q}_{28})^{(5)} + T_j^0) e^{-\left( \frac{(\hat{Q}_{28})^{(5)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{28})^{(5)} \right] \leq (\hat{Q}_{28})^{(5)}$$

In order that the operator  $\mathcal{A}^{(5)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying Into itself

The operator  $\mathcal{A}^{(5)}$  is a contraction with respect to the metric

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$$d\left( ((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t} \}$$

Indeed if we denote

$$\underline{\text{Definition of}} (\widehat{G_{31}}, \widehat{T_{31}}): (\widehat{G_{31}}, \widehat{T_{31}}) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$$

It results

$$\begin{aligned} |\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} ds_{(28)} + \\ &\int_0^t \{ (a'_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{-(\bar{M}_{28})^{(5)}s_{(28)}} + \\ &(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} + \\ &G_{28}^{(2)} | (a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) - (a''_{28})^{(5)} (T_{29}^{(2)}, s_{(28)}) | e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} \} ds_{(28)} \end{aligned}$$

Where  $s_{(28)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$|(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-(\bar{M}_{28})^{(5)}t} \leq$$

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$$\frac{1}{(\bar{M}_{28})^{(5)}} ((a_{28})^{(5)} + (a'_{28})^{(5)} + (\bar{A}_{28})^{(5)} + (\bar{P}_{28})^{(5)} (\bar{k}_{28})^{(5)}) d\left( ((G_{31})^{(1)}, (T_{31})^{(1)}); (G_{31})^{(2)}, (T_{31})^{(2)} \right)$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the THE CONCATENATED

EQUATIONS HYPOTHESISED the result follows

**Remark 1:** The fact that we supposed  $(a_{28}'')^{(5)}$  and  $(b_{28}'')^{(5)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$  and  $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$  respectively of  $\mathbb{R}_+$ .

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a_i'')^{(5)}$  and  $(b_i'')^{(5)}$ ,  $i = 28, 29, 30$  depend only on  $T_{29}$  and respectively on  $(G_{31})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$

From 19 to 28 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a_i'')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i'')^{(5)} t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{28})^{(5)})_1, ((\widehat{M}_{28})^{(5)})_2$  and  $((\widehat{M}_{28})^{(5)})_3$  :

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**Remark 3:** if  $G_{28}$  is bounded, the same property have also  $G_{29}$  and  $G_{30}$ . indeed if

$$G_{28} < ((\widehat{M}_{28})^{(5)})_1 \text{ it follows } \frac{dG_{29}}{dt} \leq ((\widehat{M}_{28})^{(5)})_1 - (a_{29}'')^{(5)} G_{29} \text{ and by integrating}$$

$$G_{29} \leq ((\widehat{M}_{28})^{(5)})_2 = G_{29}^0 + 2(a_{29})^{(5)} ((\widehat{M}_{28})^{(5)})_1 / (a_{29}'')^{(5)}$$

In the same way, one can obtain

$$G_{30} \leq ((\widehat{M}_{28})^{(5)})_3 = G_{30}^0 + 2(a_{30})^{(5)} ((\widehat{M}_{28})^{(5)})_2 / (a_{30}'')^{(5)}$$

If  $G_{29}$  or  $G_{30}$  is bounded, the same property follows for  $G_{28}$ ,  $G_{30}$  and  $G_{28}$ ,  $G_{29}$  respectively.

**Remark 4:** If  $G_{28}$  is bounded, from below, the same property holds for  $G_{29}$  and  $G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{29}$  is bounded from below.

**Remark 5:** If  $T_{28}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(5)}((G_{31})(t), t)) = (b_{29}')^{(5)}$  then  $T_{29} \rightarrow \infty$ .

**Definition of**  $(m)^{(5)}$  and  $\varepsilon_5$  :

Indeed let  $t_5$  be so that for  $t > t_5$

$$(b_{29}'')^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then  $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$  which leads to

$$T_{29} \geq \left( \frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_5 t} = \frac{1}{2} \text{ it results}$$

$$T_{29} \geq \left( \frac{(a_{29})^{(5)}(m)^{(5)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_5} \text{ By taking now } \varepsilon_5 \text{ sufficiently small one sees that } T_{29} \text{ is}$$

unbounded. The same property holds for  $T_{30}$  if  $\lim_{t \rightarrow \infty} (b_{30}'')^{(5)}((G_{31})(t), t) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of Concatenated Governing Equations Of The Totalistic System Analogous inequalities hold also for  $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take  $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$  and to choose

$(\widehat{P}_{32})^{(6)}$  and  $(\widehat{Q}_{32})^{(6)}$  large to have

$$\frac{(a_i)^{(6)}}{(\bar{M}_{32})^{(6)}} \left[ (\bar{P}_{32})^{(6)} + ((\hat{P}_{32})^{(6)} + G_j^0) e^{-\left( \frac{(\bar{P}_{32})^{(6)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{32})^{(6)}$$

$$\frac{(b_i)^{(6)}}{(\bar{M}_{32})^{(6)}} \left[ ((\hat{Q}_{32})^{(6)} + T_j^0) e^{-\left( \frac{(\hat{Q}_{32})^{(6)} + T_j^0}{T_j^0} \right)} + (\bar{Q}_{32})^{(6)} \right] \leq (\hat{Q}_{32})^{(6)}$$

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In order that the operator  $\mathcal{A}^{(6)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself

The operator  $\mathcal{A}^{(6)}$  is a contraction with respect to the metric

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$$d \left( ((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t} \}$$

Indeed if we denote

**Definition of**  $(\bar{G}_{35}, \bar{T}_{35}) : ((\bar{G}_{35}), (\bar{T}_{35})) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))$

It results

$$|\tilde{G}_{32}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} ds_{(32)} +$$

$$\int_0^t \{ (a'_{32})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{-(\bar{M}_{32})^{(6)}s_{(32)}} +$$

$$(a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} +$$

$$G_{32}^{(2)} | (a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) - (a''_{32})^{(6)} (T_{33}^{(2)}, s_{(32)}) | e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} \} ds_{(32)}$$

Where  $s_{(32)}$  represents integrand that is integrated over the interval  $[0, t]$

From the Hypothesized Governing Equations Of The Totalistic System it follows

$$|(G_{35})^{(1)} - (G_{35})^{(2)}| e^{-(\bar{M}_{32})^{(6)}t} \leq$$

$$\frac{1}{(\bar{M}_{32})^{(6)}} ((a_{32})^{(6)} + (a'_{32})^{(6)} + (\bar{A}_{32})^{(6)} + (\bar{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}) d \left( ((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)}) \right)$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a'_{32})^{(6)}$  and  $(b'_{32})^{(6)}$  depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}t}$  and  $(\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}t}$  respectively of  $\mathbb{R}_+$ .

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a'_i)^{(6)}$  and  $(b'_i)^{(6)}, i = 32, 33, 34$  depend only on  $T_{33}$  and respectively on  $(G_{35})$  (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any t where  $G_i(t) = 0$  and  $T_i(t) = 0$

From 69 to 32 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{ (a'_i)^{(6)} - (a'_i)^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \} ds_{(32)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(6)}t} > 0 \text{ for } t > 0$$

**Definition of**  $((\bar{M}_{32})^{(6)})_1, ((\bar{M}_{32})^{(6)})_2$  and  $((\bar{M}_{32})^{(6)})_3 :$

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**Remark 3:** if  $G_{32}$  is bounded, the same property have also  $G_{33}$  and  $G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$  it follows  $\frac{dG_{33}}{dt} \leq ((\widehat{M}_{32})^{(6)})_1 - (a'_{33})^{(6)} G_{33}$  and by integrating

$$G_{33} \leq ((\widehat{M}_{32})^{(6)})_2 = G_{33}^0 + 2(a_{33})^{(6)}((\widehat{M}_{32})^{(6)})_1 / (a'_{33})^{(6)}$$

In the same way, one can obtain

$$G_{34} \leq ((\widehat{M}_{32})^{(6)})_3 = G_{34}^0 + 2(a_{34})^{(6)}((\widehat{M}_{32})^{(6)})_2 / (a'_{34})^{(6)}$$

If  $G_{33}$  or  $G_{34}$  is bounded, the same property follows for  $G_{32}$ ,  $G_{34}$  and  $G_{32}$ ,  $G_{33}$  respectively.

**Remark 4:** If  $G_{32}$  is bounded, from below, the same property holds for  $G_{33}$  and  $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{33}$  is bounded from below.

**Remark 5:** If  $T_{32}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b'_i)^{(6)}((G_{35})(t), t)) = (b'_{33})^{(6)}$  then  $T_{33} \rightarrow \infty$ . 187

**Definition of**  $(m)^{(6)}$  and  $\varepsilon_6$  :

Indeed let  $t_6$  be so that for  $t > t_6$

$$(b_{33})^{(6)} - (b'_i)^{(6)}((G_{35})(t), t) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then  $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$  which leads to

$$T_{33} \geq \left( \frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6} \right) (1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_6 t} = \frac{1}{2} \text{ it results}$$

$$T_{33} \geq \left( \frac{(a_{33})^{(6)}(m)^{(6)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_6} \text{ By taking now } \varepsilon_6 \text{ sufficiently small one sees that } T_{33} \text{ is}$$

unbounded. The same property holds for  $T_{34}$  if  $\lim_{t \rightarrow \infty} (b'_{34})^{(6)}((G_{35})(t), t(t), t) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations  
OF THE GLOBAL AND UNIVERSALISTIC SYSTEM

**Behavior Of The Solutions Of Equation Representative And Constitutive Of The Totalistic And Global System:** 188

**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$  :

(a)  $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$  four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a'_{13})^{(1)}(T_{14}, t) + (a'_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b'_{13})^{(1)}(G, t) - (b'_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)}$$

(b) **Definition of**  $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$  :

(c) By  $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$  and respectively  $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$  the roots of the equations  $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$  and  $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of**  $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$  :

By  $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$  and respectively  $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$  the roots of the equations  $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$  and  $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of**  $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$  :-

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(d) If we define  $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$  by

$$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \text{ if } (v_0)^{(1)} < (v_1)^{(1)}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, \text{ if } (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$$

$$\text{and } (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \text{ if } (\bar{v}_1)^{(1)} < (v_0)^{(1)}$$

and analogously

$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \text{ if } (u_0)^{(1)} < (u_1)^{(1)}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, \text{ if } (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

$$\text{and } (u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \text{ if } (\bar{u}_1)^{(1)} < (u_0)^{(1)} \text{ where } (u_1)^{(1)}, (\bar{u}_1)^{(1)}$$

are defined

Then the solution of the GOVERNING EQUATIONS OF THE GLOBAL SYSTEM satisfies the 190  
inequalities

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$$

where  $(p_i)^{(1)}$  is defined

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left( \frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)} ((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})} \left[ e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t} \right] + G_{15}^0 e^{-(S_2)^{(1)}t} \right) \leq G_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)} ((S_1)^{(1)} - (a'_{15})^{(1)})} \left[ e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t} \right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)} ((R_1)^{(1)} - (b'_{15})^{(1)})} \left[ e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)} ((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[ e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t} \right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of**  $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$ :-

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Where  $(S_1)^{(1)} = (a_{13})^{(1)} (m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)} (\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

**Behavior Of The Solutions Of Equation Hypothesizing The Globality Of The System And Consequential Concatenated Form:**

**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$  :

(e)  $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$  four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)} (T_{17}, t) + (a''_{17})^{(2)} (T_{17}, t) \leq -(\sigma_1)^{(2)}$$



$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)}$$

**Definition of**  $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$  :

By  $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$  and respectively  $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$  the roots

$$(f) \quad \text{of the equations } (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$$

$$\text{and } (b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0 \text{ and}$$

**Definition of**  $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$  :

By  $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$  and respectively  $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$  the

$$\text{roots of the equations } (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$$

$$\text{and } (b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$$

**Definition of**  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  :-

(g) If we define  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  by

$$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)},$$

$$\text{and } (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (\bar{v}_1)^{(2)} < (v_0)^{(2)}$$

and analogously

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

$$\text{and } (u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)}$$

Then the solution of the system satisfies the inequalities

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$  is defined

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$$

$$\left( \frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[ e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})} \left[ e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})} \left[ e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[ e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

**Definition of**  $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$  :-

$$\begin{aligned} \text{Where } (S_1)^{(2)} &= (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)} \\ (S_2)^{(2)} &= (a_{18})^{(2)} - (p_{18})^{(2)} \\ (R_1)^{(2)} &= (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \\ (R_2)^{(2)} &= (b'_{18})^{(2)} - (r_{18})^{(2)} \end{aligned}$$

### **Behavior of the solutions of equation 37 to 42**

**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$  :

(a)  $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$  four constants satisfying

$$\begin{aligned} -(\sigma_2)^{(3)} &\leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)} \\ -(\tau_2)^{(3)} &\leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)} \end{aligned}$$

**Definition of**  $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$  :

(b) By  $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$  and respectively  $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$  the roots of the equations  $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and  $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$  and

By  $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$  and respectively  $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$  the

roots of the equations  $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and  $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of**  $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$  :-

(c) If we define  $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$  by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)},$

and  $(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}, \text{ and } (u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \text{ if } (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of **The Global Concatenated Equations** satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$  is defined ABOVE

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$$

$$\left( \frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})} \left[ e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \leq G_{22}(t) \leq \right.$$

$$\frac{(a_{22})^{(3)}G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)}-(a'_{22})^{(3)})} [e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G_{22}^0 e^{-(a'_{22})^{(3)}t}$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}}$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})} [e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \leq T_{22}(t) \leq$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})} [e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

**Definition of**  $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$ :-

Where  $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

### **Behavior of the solutions**

**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$  :

(d)  $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$  four constants satisfying

$$-(\sigma_2)^{(4)} \leq -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \leq -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \leq -(\tau_1)^{(4)}$$

**Definition of**  $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$  :

(e) By  $(v_1)^{(4)} > 0, (v_2)^{(4)} < 0$  and respectively  $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$  the roots of the equations  $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and  $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$  and

**Definition of**  $(\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$  :

By  $(\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0$  and respectively  $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$  the

roots of the equations  $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and  $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of**  $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$  :-

(f) If we define  $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$  by

$$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}, \text{ if } (v_0)^{(4)} < (v_1)^{(4)}$$

$$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}, \text{ if } (v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}, \text{ and } \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$$

$$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}, \text{ if } (\bar{v}_4)^{(4)} < (v_0)^{(4)}$$

and analogously

$$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \text{ if } (u_0)^{(4)} < (u_1)^{(4)}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \text{ if } (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}, \text{ and } \boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \text{ if } (\bar{u}_1)^{(4)} < (u_0)^{(4)} \text{ where } (u_1)^{(4)}, (\bar{u}_1)^{(4)}$$

Then the solution of GLOBAL EQUATIONS satisfies the inequalities

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$

where  $(p_i)^{(4)}$  is defined ABOVE

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

$$\left( \frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)} ((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t} \right] + G_{26}^0 e^{-(S_2)^{(4)}t} \leq G_{26}(t) \leq \right. \\ \left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)} ((S_1)^{(4)} - (a'_{26})^{(4)})} \left[ e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t} \right] + G_{26}^0 e^{-(a'_{26})^{(4)}t} \right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)} ((R_1)^{(4)} - (b'_{26})^{(4)})} \left[ e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \leq T_{26}(t) \leq$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)} ((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[ e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t} \right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of**  $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$ :-

Where  $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$$

$$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

**Behavior of the solutions of GLOBAL EQUATIONS:**

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If we denote and define

**Definition of**  $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$  :

(g)  $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$  four constants satisfying

$$-(\sigma_2)^{(5)} \leq -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \leq -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \leq -(\tau_1)^{(5)}$$

**Definition of**  $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$  :

(h) By  $(v_1)^{(5)} > 0, (v_2)^{(5)} < 0$  and respectively  $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$  the roots of the equations  $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$

and  $(b_{29})^{(5)}(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$  and

**Definition of**  $(\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$  :

By  $(\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0$  and respectively  $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$  the

roots of the equations  $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$

and  $(b_{29})^{(5)}(u^{(5)})^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of**  $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$  :-

(i) If we define  $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$  by

$$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \text{ if } (v_0)^{(5)} < (v_1)^{(5)}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, \text{ if } (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}, \text{ and } \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \text{ if } (\bar{v}_1)^{(5)} < (v_0)^{(5)}$$

and analogously

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \text{ if } (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \text{ if } (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}, \text{ and } \boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \text{ if } (\bar{u}_1)^{(5)} < (u_0)^{(5)} \text{ where } (u_1)^{(5)}, (\bar{u}_1)^{(5)} \text{ are defined ABOVE}$$

Then the solution of THE CONCATENATED GLOBAL EQUATIONS satisfies the inequalities

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t}$$

where  $(p_i)^{(5)}$  is defined ABOVE

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t}$$

$$\left( \frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)} ((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \right) \left[ e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t} \right] + G_{30}^0 e^{-(S_2)^{(5)}t} \leq G_{30}(t) \leq$$

$$\frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)} ((S_1)^{(5)} - (a'_{30})^{(5)})} \left[ e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)}t}$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)} ((R_1)^{(5)} - (b'_{30})^{(5)})} \left[ e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \leq T_{30}(t) \leq$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)} ((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[ e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t} \right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

**Definition of**  $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$ :-

Where  $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$$

$$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$$

$$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$$

**Behavior Of The Solutions Of Equation Constitutive Of Global Status Of The System (Module Six):**

If we denote and define

**Definition of**  $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$  :

(j)  $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$  four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

**Definition of**  $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$  :

(k) By  $(v_1)^{(6)} > 0, (v_2)^{(6)} < 0$  and respectively  $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$  the roots of the equations  $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and  $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$  and

**Definition of**  $(\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$  :

By  $(\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0$  and respectively  $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$  the

roots of the equations  $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and  $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of**  $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}$  :-

(1) If we define  $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$  by

$$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}, \text{ and } (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (\bar{v}_1)^{(6)} < (v_0)^{(6)}$$

and analogously

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}, \text{ and } (u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)} \text{ are defined ABOVE}$$

Then the solution of THE GLOBAL CONCATENATED EQUATIONS satisfies the inequalities

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$

where  $(p_i)^{(6)}$  is defined IN THE FOREGOING

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t}$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)} ((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t} \right] + G_{34}^0 e^{-(S_2)^{(6)}t} \right) \leq G_{34}(t) \leq$$

$$\frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)} ((S_1)^{(6)} - (a'_{34})^{(6)})} \left[ e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}}$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)} ((R_1)^{(6)} - (b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \leq T_{34}(t) \leq$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)} ((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[ e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t} \right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

**Definition of**  $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$  :-

Where  $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

## MODULE ONE

**Proof:** From THE GLOBAL EQUATIONS CONCATENATED FOR MODULE ONE we obtain

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left( (a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of**  $v^{(1)}$  :- 
$$v^{(1)} = \frac{G_{13}}{G_{14}}$$

It follows

$$- \left( (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left( (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$  :-

(a) For  $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_0)^{(1)})t]}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_0)^{(1)})t]}} , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

$$\text{it follows } (v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$$

In the same manner , we get

$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}} , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

From which we deduce  $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(b) If  $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)}$  we find like in the previous case,

$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t]}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t]}} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}} \leq (\bar{v}_1)^{(1)}$$

(c) If  $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$  , we obtain

$$(v_1)^{(1)} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}} \leq (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of**  $v^{(1)}(t)$  :-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(1)}(t)$  :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in FIRST MODULE OF THE CONCATENATED SYSTEM OF GLOBAL SYSTEM we get easily the result stated in the theorem.

Particular case :

If  $(a''_{13})^{(1)} = (a''_{14})^{(1)}$ , then  $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$  and in this case  $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$  if in addition

$(v_0)^{(1)} = (v_1)^{(1)}$  then  $v^{(1)}(t) = (v_0)^{(1)}$  and as a consequence  $G_{13}(t) = (v_0)^{(1)}G_{14}(t)$  this also defines  $(v_0)^{(1)}$  for the special case

Analogously if  $(b''_{13})^{(1)} = (b''_{14})^{(1)}$ , then  $(\tau_1)^{(1)} = (\tau_2)^{(1)}$  and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$  if in addition  $(u_0)^{(1)} = (u_1)^{(1)}$  then  $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$  This is an important consequence of the relation between  $(v_1)^{(1)}$  and  $(\bar{v}_1)^{(1)}$ , and definition of  $(u_0)^{(1)}$ .

## MODULE NUMBERED TWO

**Proof:** From GLOBAL EQUATIONS CONCATENATED SYSTEM we obtain

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left( (a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

**Definition of**  $v^{(2)}$  :- 
$$v^{(2)} = \frac{G_{16}}{G_{17}}$$

It follows

$$- \left( (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} \right) \leq \frac{dv^{(2)}}{dt} \leq - \left( (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$  :-

(d) For  $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}}{1 + (C)^{(2)}e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}} , \quad (C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}$$

it follows  $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner, we get

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} , \quad (\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}$$

From which we deduce  $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$

(e) If  $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$  we find like in the previous case,

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}}{1 + (C)^{(2)}e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (\bar{v}_1)^{(2)}$$

(f) If  $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(2)}(t)$  :-

$$(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)}, \quad v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(2)}(t)$  :-



$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in the system equations we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{16})^{(2)} = (a''_{17})^{(2)}$ , then  $(\sigma_1)^{(2)} = (\sigma_2)^{(2)}$  and in this case  $(v_1)^{(2)} = (\bar{v}_1)^{(2)}$  if in addition  $(v_0)^{(2)} = (v_1)^{(2)}$  then  $v^{(2)}(t) = (v_0)^{(2)}$  and as a consequence  $G_{16}(t) = (v_0)^{(2)}G_{17}(t)$

Analogously if  $(b''_{16})^{(2)} = (b''_{17})^{(2)}$ , then  $(\tau_1)^{(2)} = (\tau_2)^{(2)}$  and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$  if in addition  $(u_0)^{(2)} = (u_1)^{(2)}$  then  $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$  This is an important consequence of the relation between  $(v_1)^{(2)}$  and  $(\bar{v}_1)^{(2)}$

**MODULE BEARING NUMBER THREE**

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left( (a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) \right) - (a''_{21})^{(3)}(T_{21}, t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

**Definition of**  $v^{(3)}$  :-  $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$

It follows

$$- \left( (a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} \right) \leq \frac{dv^{(3)}}{dt} \leq - \left( (a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} \right)$$

From which one obtains

(a) For  $0 < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_0)^{(3)})t]}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_0)^{(3)})t]}} , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows  $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner, we get

$$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

**Definition of**  $(\bar{v}_1)^{(3)}$  :-

From which we deduce  $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

(b) If  $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{v}_1)^{(3)}$  we find like in the previous case,

$$(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_2)^{(3)})t]}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_2)^{(3)})t]}} \leq v^{(3)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} \leq (\bar{v}_1)^{(3)}$$

(c) If  $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain

$$(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} \leq (v_0)^{(3)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(3)}(t)$  :-

$$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(3)}(t)$  :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in the system equations we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{20})^{(3)} = (a''_{21})^{(3)}$ , then  $(\sigma_1)^{(3)} = (\sigma_2)^{(3)}$  and in this case  $(v_1)^{(3)} = (\bar{v}_1)^{(3)}$  if in addition  $(v_0)^{(3)} = (v_1)^{(3)}$  then  $v^{(3)}(t) = (v_0)^{(3)}$  and as a consequence  $G_{20}(t) = (v_0)^{(3)}G_{21}(t)$

Analogously if  $(b''_{20})^{(3)} = (b''_{21})^{(3)}$ , then  $(\tau_1)^{(3)} = (\tau_2)^{(3)}$  and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$  if in addition  $(u_0)^{(3)} = (u_1)^{(3)}$  then  $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$  This is an important consequence of the relation between  $(v_1)^{(3)}$  and  $(\bar{v}_1)^{(3)}$

MODULE BEARING NUMBER FOUR IN THE CONCATENATED GLOBAL SYSTEM

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left( (a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) \right) - (a''_{25})^{(4)}(T_{25}, t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of**  $v^{(4)}$  :-  $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$- \left( (a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} \right) \leq \frac{dv^{(4)}}{dt} \leq - \left( (a_{25})^{(4)}(v^{(4)})^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(4)}, (v_0)^{(4)}$  :-

(d) For  $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}}{4 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(v_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}}$$

it follows  $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner, we get

$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{4 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}}$$

From which we deduce  $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

(e) If  $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$  we find like in the previous case,

$$(v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_2)^{(4)})t]}}{1 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_2)^{(4)})t]}} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \leq (\bar{v}_1)^{(4)}$$

(f) If  $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(4)}(t)$  :-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(4)}(t)$  :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in THE CONCATENATED SYSTEM OF THE GLOBAL ORDER we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{24})^{(4)} = (a''_{25})^{(4)}$ , then  $(\sigma_1)^{(4)} = (\sigma_2)^{(4)}$  and in this case  $(v_1)^{(4)} = (\bar{v}_1)^{(4)}$  if in addition  $(v_0)^{(4)} = (v_1)^{(4)}$  then  $v^{(4)}(t) = (v_0)^{(4)}$  and as a consequence  $G_{24}(t) = (v_0)^{(4)}G_{25}(t)$  **this also defines**  $(v_0)^{(4)}$  **for the special case**.

Analogously if  $(b''_{24})^{(4)} = (b''_{25})^{(4)}$ , then  $(\tau_1)^{(4)} = (\tau_2)^{(4)}$  and then

$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$  if in addition  $(u_0)^{(4)} = (u_1)^{(4)}$  then  $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$  This is an important consequence of the relation between  $(v_1)^{(4)}$  and  $(\bar{v}_1)^{(4)}$ , **and definition of**  $(u_0)^{(4)}$ .

MODULE BEARING NUMBER FIVE IN THE GLOBAL EQUATIONS WHICH ARE CONCATENATED THE FOLLOWING NATURALLY HOLDS AND IS PROVED.

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left( (a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right) - (a'_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

**Definition of**  $v^{(5)}$  :-  $\boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$- \left( (a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} \right) \leq \frac{dv^{(5)}}{dt} \leq - \left( (a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$  :-

(g) For  $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}}{5 + (C)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}} , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}}$$

it follows  $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner, we get

$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{5 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}}$$

From which we deduce  $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

(h) If  $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{v}_1)^{(5)}$  we find like in the previous case,

$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (\bar{c})^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_2)^{(5)})t]}}{1 + (\bar{c})^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_2)^{(5)})t]}} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{c})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{c})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \leq (\bar{v}_1)^{(5)}$$

(i) If  $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain

$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{c})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{c})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(5)}(t)$  :-

$$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(5)}(t)$  :-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}$ , then  $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$  and in this case  $(v_1)^{(5)} = (\bar{v}_1)^{(5)}$  if in addition  $(v_0)^{(5)} = (v_5)^{(5)}$  then  $v^{(5)}(t) = (v_0)^{(5)}$  and as a consequence  $G_{28}(t) = (v_0)^{(5)}G_{29}(t)$  **this also defines  $(v_0)^{(5)}$  for the special case .**

Analogously if  $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}$ , then  $(\tau_1)^{(5)} = (\tau_2)^{(5)}$  and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$  if in addition  $(u_0)^{(5)} = (u_1)^{(5)}$  then  $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$  This is an important consequence of the relation between  $(v_1)^{(5)}$  and  $(\bar{v}_1)^{(5)}$ , **and definition of  $(u_0)^{(5)}$ .**

MODULE NUMBERED SIX IN THE CONCATENATED GLOBAL EQUATIONS OBTAINED CONSEQUENTIAL TO THE CONCATENATION PROCESS

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left( (a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t) \right) - (a_{33}'')^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

**Definition of**  $v^{(6)}$  :-  $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$- \left( (a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} \right) \leq \frac{dv^{(6)}}{dt} \leq - \left( (a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$  :-

(j) For  $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_0)^{(6)})t]}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_0)^{(6)})t]}} , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}}$$

it follows  $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner, we get

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}} , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}}$$

From which we deduce  $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

(k) If  $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$  we find like in the previous case,

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_2)^{(6)})t]}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_2)^{(6)})t]}} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}} \leq (\bar{v}_1)^{(6)}$$

(l) If  $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(6)}(t)$  :-

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(6)}(t)$  :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

#### **Particular case :FOR THE ENTIRE GLOBAL SYTEM WITH RESPECT TO MODULE SIX**

If  $(a''_{32})^{(6)} = (a''_{33})^{(6)}$ , then  $(\sigma_1)^{(6)} = (\sigma_2)^{(6)}$  and in this case  $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$  if in addition  $(v_0)^{(6)} = (v_1)^{(6)}$  then  $v^{(6)}(t) = (v_0)^{(6)}$  and as a consequence  $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$  **this also defines  $(v_0)^{(6)}$  for the special case .**

Analogously if  $(b''_{32})^{(6)} = (b''_{33})^{(6)}$ , then  $(\tau_1)^{(6)} = (\tau_2)^{(6)}$  and then

$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$  if in addition  $(u_0)^{(6)} = (u_1)^{(6)}$  then  $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$  This is an important consequence of the relation between  $(v_1)^{(6)}$  and  $(\bar{v}_1)^{(6)}$ , **and definition of  $(u_0)^{(6)}$ .**

We can prove the following FOR THE CONCATENATED SYSTEM OF EQUATIONS FOR THE GLOBAL ORDER(FIRST MODULE TO SIXTH MODULE)

**Theorem** If  $(a'_i)^{(1)}$  and  $(b'_i)^{(1)}$  are independent on  $t$ , and the conditions

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0 ,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with  $(p_{13})^{(1)}, (r_{14})^{(1)}$  as defined ABOVE are satisfied , then the system

SECOND MODULE OF QUANTUM COMOPUTING AND QUANTUM ADVICE IN THE CONCATENATED EQUATIONS HAS TO SATISFY IN THE HOLISTIC EQUATIONAL ORDER:

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0 ,$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$$

with  $(p_{16})^{(2)}, (r_{17})^{(2)}$  as defined ABOVE are satisfied , then the system

**Theorem** If  $(a''_i)^{(3)}$  and  $(b''_i)^{(3)}$  are independent on  $t$  , and the conditions

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0 ,$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

with  $(p_{20})^{(3)}, (r_{21})^{(3)}$  satisfied , then the system

We can prove the following

If  $(a''_i)^{(4)}$  and  $(b''_i)^{(4)}$  are independent on  $t$  , and the conditions

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$$

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0 ,$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$$

with  $(p_{24})^{(4)}, (r_{25})^{(4)}$  as defined by equation are satisfied , then the system

If  $(a''_i)^{(5)}$  and  $(b''_i)^{(5)}$  are independent on  $t$  , and the conditions

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$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$$

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0 ,$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$$

with  $(p_{28})^{(5)}, (r_{29})^{(5)}$  as defined ABOVE are satisfied , then the system

If  $(a''_i)^{(6)}$  and  $(b''_i)^{(6)}$  are independent on  $t$  , and the conditions

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$$

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0 ,$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$$

with  $(p_{32})^{(6)}, (r_{33})^{(6)}$  as defined ABOVE are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0$$

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$$(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0 \quad 196$$

$$(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0 \quad 197$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0 \quad 198$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0 \quad 199$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0 \quad 200$$

has a unique positive solution , which is an equilibrium solution for the system (GLOBAL SYSTEM)

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0 \quad 201$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0 \quad 202$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0 \quad 203$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0 \quad 204$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0 \quad 205$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0 \quad 206$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0 \quad 207$$

$$(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0 \quad 208$$

$$(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0 \quad 209$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0 \quad 210$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0 \quad 211$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0 \quad 212$$

has a unique positive solution , which is an equilibrium solution for THE GLOBAL EQUATIONS

$$(a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} = 0 \quad 213$$

$$(a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} = 0 \quad 214$$

$$(a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} = 0 \quad 215$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}))]T_{24} = 0 \quad 216$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}))]T_{25} = 0 \quad 217$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}))]T_{26} = 0 \quad 218$$

has a unique positive solution , which is an equilibrium solution for the system WHICH IS HOLISTIC  
DEFINED BY THE CONCATENATED SYSTEM OF EQUATIONS WHICH ARE CONSEQUENTIAL TO THE  
MODULE EQUATIONS

$$(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0 \quad 219$$

$$(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0 \quad 220$$

$$(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0 \quad 221$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 222$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 223$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 224$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0 \quad 225$$

$$(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} = 0 \quad 226$$

$$(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} = 0 \quad 227$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 228$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 229$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 230$$

has a unique positive solution , which is an equilibrium solution for the system

Indeed the first two equations have a nontrivial solution  $G_{13}, G_{14}$  if FOR SOAP BUBBLE AND PROTEIN FOLDING

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Indeed the first two equations have a nontrivial solution  $G_{16}, G_{17}$  if FOR QUANTUM COMPUTING AND QUANTUM ADVICE

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

(a) Indeed the first two equations have a nontrivial solution  $G_{20}, G_{21}$  if FOR THE QUANTUM ADIABATIC ALGORITHMS AND QUANTUM MECHANICAL NONLINEARITIES

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

(a) Indeed the first two equations have a nontrivial solution  $G_{24}, G_{25}$  if HIDDEN VARIABLES AND RELATIVISTIC TIME DILATION

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

(a) Indeed the first two equations have a nontrivial solution  $G_{28}, G_{29}$  if FOR ANALOG COMPUTING AND MALAMENT HOGARTH SPACE TIMES

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

(a) Indeed the first two equations have a nontrivial solution  $G_{32}, G_{33}$  if FOR QUANTUM GRAVITY AND ANTHROPIC COMPUTING

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Definition and uniqueness of  $T_{14}^*$  :-**

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a''_i)^{(1)}(T_{14})$  being increasing, it follows that there exists a unique  $T_{14}^*$  for which  $f(T_{14}^*) = 0$ . With this value , we obtain from the three first equations



$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$$

**Definition and uniqueness of  $T_{17}^*$  :-**

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(2)}(T_{17})$  being increasing, it follows that there exists a unique  $T_{17}^*$  for which  $f(T_{17}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]}$$

**Definition and uniqueness of  $T_{21}^*$  :-**

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(1)}(T_{21})$  being increasing, it follows that there exists a unique  $T_{21}^*$  for which  $f(T_{21}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

**Definition and uniqueness of  $T_{25}^*$  :-**

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(4)}(T_{25})$  being increasing, it follows that there exists a unique  $T_{25}^*$  for which  $f(T_{25}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

**Definition and uniqueness of  $T_{29}^*$  :-**

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(5)}(T_{29})$  being increasing, it follows that there exists a unique  $T_{29}^*$  for which  $f(T_{29}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

**Definition and uniqueness of  $T_{33}^*$  :-**

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(6)}(T_{33})$  being increasing, it follows that there exists a unique  $T_{33}^*$  for which  $f(T_{33}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

(e) By the same argument, the equations FOR THE GLOBAL SYSTEM admit solutions  $G_{13}, G_{14}$  if

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - \\ [(b'_{13})^{(1)}(b'_{14})^{(1)}(G) + (b'_{14})^{(1)}(b'_{13})^{(1)}(G)] + (b'_{13})^{(1)}(G)(b'_{14})^{(1)}(G) = 0$$

Where in  $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{14}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{14}^*$  such that  $\varphi(G^*) = 0$

(f) By the same argument, the equations FOR THE GLOBAL SYSTEM admit solutions  $G_{16}, G_{17}$  if

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b'_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in  $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{17}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{14}^*$  such that  $\varphi((G_{19})^*) = 0$

(g) By the same argument, the equations FOR THE GLOBAL SYSTEM admit solutions  $G_{20}, G_{21}$  if

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b'_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in  $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{21}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{21}^*$  such that  $\varphi((G_{23})^*) = 0$

(h) By the same argument, the equations FOR GLOBAL SYSTEM admit solutions  $G_{24}, G_{25}$  if

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})] + (b'_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in  $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{25}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{25}^*$  such that  $\varphi((G_{27})^*) = 0$

(i) By the same argument, the equations FOR GLOBAL SYSTEM admit solutions  $G_{28}, G_{29}$  if

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b'_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in  $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{29}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{29}^*$  such that  $\varphi((G_{31})^*) = 0$

(j) By the same argument, the equations FOR GLOBAL SYSTEM admit solutions  $G_{32}, G_{33}$  if

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})] + (b'_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in  $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{33}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{33}^*$  such that  $\varphi(G^*) = 0$

Finally we obtain the unique solution of the CONSEQUENTIAL CONCATENATED EQUATIONS OF THE HOLISTIC TOTALISTIC SYSTEM

$G_{14}^*$  given by  $\varphi(G^*) = 0, T_{14}^*$  given by  $f(T_{14}^*) = 0$  and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]}, \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]}, \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution of GLOBAL EQUATIONS

Finally we obtain the unique solution of GLOBAL EQUATION OF THE SYSTEM:

$G_{17}^*$  given by  $\varphi((G_{19})^*) = 0$ ,  $T_{17}^*$  given by  $f(T_{17}^*) = 0$  and

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]}$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]}$$

Obviously, these values represent an equilibrium solution of GLOBAL SYSTEM

Finally we obtain the unique solution of the GLOBAL GOVERNING EQUATIONS

$G_{21}^*$  given by  $\varphi((G_{23})^*) = 0$ ,  $T_{21}^*$  given by  $f(T_{21}^*) = 0$  and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}((G_{23})^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}((G_{23})^*)]}$$

Obviously, these values represent an equilibrium solution of GOVERNING GLOBAL EQUATIONS

Finally we obtain the unique solution of GLOBAL EQUATIONS

$G_{25}^*$  given by  $\varphi(G_{27}) = 0$ ,  $T_{25}^*$  given by  $f(T_{25}^*) = 0$  and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]}$$

Obviously, these values represent an equilibrium solution of GOVERNING GLOBAL EQUATIONS

Finally we obtain the unique solution of GOVERNING GLOBAL EQUATIONS

$G_{29}^*$  given by  $\varphi((G_{31})^*) = 0$ ,  $T_{29}^*$  given by  $f(T_{29}^*) = 0$  and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]}$$

Obviously, these values represent an equilibrium solution of GOVERNING GLOBAL EQUATIONS,

Finally we obtain the unique solution of CONCATENATED SYSTEM OF GLOBAL EQUATIONS

$G_{33}^*$  given by  $\varphi((G_{35})^*) = 0$ ,  $T_{33}^*$  given by  $f(T_{33}^*) = 0$  and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]}$$

Obviously, these values represent an equilibrium solution of GLOBAL EQUATIONS

#### ASYMPTOTIC STABILITY ANALYSIS

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(1)}$  and  $(b_i'')^{(1)}$  belong to  $C^{(1)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations PERTAINING TO THE GLOBAL SYSTEM IN QUESTION and neglecting the terms of power 2, we obtain

$$\begin{aligned}\frac{d\mathbb{G}_{13}}{dt} &= -((a'_{13})^{(1)} + (p_{13})^{(1)})\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \\ \frac{d\mathbb{G}_{14}}{dt} &= -((a'_{14})^{(1)} + (p_{14})^{(1)})\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \\ \frac{d\mathbb{G}_{15}}{dt} &= -((a'_{15})^{(1)} + (p_{15})^{(1)})\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \\ \frac{d\mathbb{T}_{13}}{dt} &= -((b'_{13})^{(1)} - (r_{13})^{(1)})\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}(s_{(13)(j)})T_{13}^*\mathbb{G}_j \\ \frac{d\mathbb{T}_{14}}{dt} &= -((b'_{14})^{(1)} - (r_{14})^{(1)})\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}(s_{(14)(j)})T_{14}^*\mathbb{G}_j \\ \frac{d\mathbb{T}_{15}}{dt} &= -((b'_{15})^{(1)} - (r_{15})^{(1)})\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}(s_{(15)(j)})T_{15}^*\mathbb{G}_j\end{aligned}$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(2)}$  and  $(b_i'')^{(2)}$  belong to  $C^{(2)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable (FOR QUANTUM COMPUTING AND QUANTUM ADVICE)

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :-

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)}, \quad \frac{\partial(b_i'')^{(2)}}{\partial G_j}(G_{19}^*) = s_{ij}$$

Taking into account equations PERTAINING TO THE GLOBAL SYSTEM and neglecting the terms of power 2, we obtain FOR THE GLOBAL SYSTEM, AS THE CONTRIBUTION FROM THE MODULE OF QUANTUM COMPUTING AND QUANTUM ADVICE. IT IS NECESSARY THAT THE MODULES MUST BE BORNE IN MINS AND WE SHALL NOT REPEAT THIS EXPRESSIVELY IN THE WORK.

$$\frac{d\mathbb{G}_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)})\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 231$$

$$\frac{d\mathbb{G}_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)})\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 232$$

$$\frac{d\mathbb{G}_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)})\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \quad 233$$

$$\frac{d\mathbb{T}_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}(s_{(16)(j)})T_{16}^*\mathbb{G}_j \quad 234$$

$$\frac{d\mathbb{T}_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}(s_{(17)(j)})T_{17}^*\mathbb{G}_j \quad 235$$

$$\frac{d\mathbb{T}_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}(s_{(18)(j)})T_{18}^*\mathbb{G}_j \quad 236$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(3)}$  and  $(b_i'')^{(3)}$  belong to  $C^{(3)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable. (THIRD MODULE CONTRIBUTION)

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :-

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)}, \quad \frac{\partial(b_i'')^{(3)}}{\partial G_j}(G_{23}^*) = s_{ij}$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 237$$

$$\frac{d\mathbb{G}_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)})\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 238$$

$$\frac{d\mathbb{G}_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)})\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 239$$

$$\frac{d\mathbb{T}_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)})\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}(s_{(20)(j)})T_{20}^*\mathbb{G}_j \quad 240$$

$$\frac{d\mathbb{T}_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)})\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}(s_{(21)(j)})T_{21}^*\mathbb{G}_j \quad 241$$

$$\frac{d\mathbb{T}_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)})\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}(s_{(22)(j)})T_{22}^*\mathbb{G}_j \quad 242$$

#### FOURTH MODULE CONTRIBUTION TO THE GLOBAL EQUATIONS

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(4)}$  and  $(b_i'')^{(4)}$  belong to  $C^{(4)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :-

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}, \quad \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$

Then taking into account equations GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -((a'_{24})^{(4)} + (p_{24})^{(4)})\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 243$$

$$\frac{d\mathbb{G}_{25}}{dt} = -((a'_{25})^{(4)} + (p_{25})^{(4)})\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 244$$

$$\frac{d\mathbb{G}_{26}}{dt} = -((a'_{26})^{(4)} + (p_{26})^{(4)})\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 245$$

$$\frac{d\mathbb{T}_{24}}{dt} = -((b'_{24})^{(4)} - (r_{24})^{(4)})\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}(s_{(24)(j)})T_{24}^*\mathbb{G}_j \quad 246$$

$$\frac{d\mathbb{T}_{25}}{dt} = -((b'_{25})^{(4)} - (r_{25})^{(4)})\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}(s_{(25)(j)})T_{25}^*\mathbb{G}_j \quad 247$$

$$\frac{d\mathbb{T}_{26}}{dt} = -((b'_{26})^{(4)} - (r_{26})^{(4)})\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}(s_{(26)(j)})T_{26}^*\mathbb{G}_j \quad 248$$

#### A CONTRIBUTION TO THE HOLISTIC SYSTEMAL EQUATIONS FROM THE FIFTH MODULE NAMELY ANALOG COMPUTING AND MALAMENT HOGHWART SPACETIMES

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(5)}$  and  $(b_i'')^{(5)}$  belong to  $C^{(5)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :-

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}, \quad \frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations PERTAINING BTO THE GLOBAL FIELD and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -((a'_{28})^{(5)} + (p_{28})^{(5)})\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 249$$

$$\frac{d\mathbb{G}_{29}}{dt} = -((a'_{29})^{(5)} + (p_{29})^{(5)})\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 250$$

$$\frac{d\mathbb{G}_{30}}{dt} = -((a'_{30})^{(5)} + (p_{30})^{(5)})\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*T_{29} \quad 251$$

$$\frac{dT_{28}}{dt} = -((b'_{28})^{(5)} - (r_{28})^{(5)})T_{28} + (b_{28})^{(5)}T_{29} + \sum_{j=28}^{30}(s_{(28)(j)})T_{28}^*\mathbb{G}_j \quad 252$$

$$\frac{dT_{29}}{dt} = -((b'_{29})^{(5)} - (r_{29})^{(5)})T_{29} + (b_{29})^{(5)}T_{28} + \sum_{j=28}^{30}(s_{(29)(j)})T_{29}^*\mathbb{G}_j \quad 253$$

$$\frac{dT_{30}}{dt} = -((b'_{30})^{(5)} - (r_{30})^{(5)})T_{30} + (b_{30})^{(5)}T_{29} + \sum_{j=28}^{30}(s_{(30)(j)})T_{30}^*\mathbb{G}_j \quad 254$$

#### A SIXTH MODULE CONTRIBUTION(QUANTUM GRAVITY AND ANTHROPIC COMPUTING)

If the conditions of the previous theorem are satisfied and if the functions  $(a'_i)^{(6)}$  and  $(b'_i)^{(6)}$  Belong to  $C^{(6)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of**  $\mathbb{G}_i, T_i$  :-

$$G_i = G_i^* + \mathbb{G}_i, T_i = T_i^* + T_i$$

$$\frac{\partial(a'_{33})^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}, \frac{\partial(b'_i)^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{32}}{dt} = -((a'_{32})^{(6)} + (p_{32})^{(6)})\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*T_{33} \quad 255$$

$$\frac{d\mathbb{G}_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*T_{33} \quad 256$$

$$\frac{d\mathbb{G}_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*T_{33} \quad 257$$

$$\frac{dT_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})T_{32} + (b_{32})^{(6)}T_{33} + \sum_{j=32}^{34}(s_{(32)(j)})T_{32}^*\mathbb{G}_j \quad 258$$

$$\frac{dT_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})T_{33} + (b_{33})^{(6)}T_{32} + \sum_{j=32}^{34}(s_{(33)(j)})T_{33}^*\mathbb{G}_j \quad 259$$

$$\frac{dT_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})T_{34} + (b_{34})^{(6)}T_{33} + \sum_{j=32}^{34}(s_{(34)(j)})T_{34}^*\mathbb{G}_j \quad 260$$

The characteristic equation of this system is

$$\begin{aligned} &((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)})\{((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}) \\ &\left[ \left( ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)})(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^* \right) \right] \\ &\left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^* \right) \\ &+ \left( ((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)})(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^* \right) \\ &\left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^* \right) \\ &\left( ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) \\ &\left( ((\lambda)^{(1)})^2 + ((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}) (\lambda)^{(1)} \right) \\ &+ \left( ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) (q_{15})^{(1)}G_{15} \\ &+ ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) ((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*) \\ &\left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^* \right) \} = 0 \\ &+ \\ &((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)})\{((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}) \end{aligned} \quad 261$$

$$\begin{aligned}
 & \left[ \left( (\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)} \right) (q_{17})^{(2)} G_{17}^* + (a_{17})^{(2)} (q_{16})^{(2)} G_{16}^* \right] \\
 & \left( (\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)} \right) s_{(17),(17)} T_{17}^* + (b_{17})^{(2)} s_{(16),(17)} T_{17}^* \\
 & + \left( (\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)} \right) (q_{16})^{(2)} G_{16}^* + (a_{16})^{(2)} (q_{17})^{(2)} G_{17}^* \\
 & \left( (\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)} \right) s_{(17),(16)} T_{17}^* + (b_{17})^{(2)} s_{(16),(16)} T_{16}^* \\
 & \left( (\lambda)^{(2)} \right)^2 + \left( (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (\lambda)^{(2)} \\
 & \left( (\lambda)^{(2)} \right)^2 + \left( (b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)} \right) (\lambda)^{(2)} \\
 & + \left( (\lambda)^{(2)} \right)^2 + \left( (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (\lambda)^{(2)} (q_{18})^{(2)} G_{18} \\
 & + \left( (\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)} \right) \left( (a_{18})^{(2)} (q_{17})^{(2)} G_{17}^* + (a_{17})^{(2)} (a_{18})^{(2)} (q_{16})^{(2)} G_{16}^* \right) \\
 & \left( (\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)} \right) s_{(17),(18)} T_{17}^* + (b_{17})^{(2)} s_{(16),(18)} T_{16}^* \} = 0 \\
 & + \\
 & \left( (\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)} \right) \{ (\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)} \} \\
 & \left[ \left( (\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)} \right) (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (q_{20})^{(3)} G_{20}^* \right] \\
 & \left( (\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(21)} T_{21}^* + (b_{21})^{(3)} s_{(20),(21)} T_{21}^* \\
 & + \left( (\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)} \right) (q_{20})^{(3)} G_{20}^* + (a_{20})^{(3)} (q_{21})^{(1)} G_{21}^* \\
 & \left( (\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(20)} T_{21}^* + (b_{21})^{(3)} s_{(20),(20)} T_{20}^* \\
 & \left( (\lambda)^{(3)} \right)^2 + \left( (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} \\
 & \left( (\lambda)^{(3)} \right)^2 + \left( (b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)} \right) (\lambda)^{(3)} \\
 & + \left( (\lambda)^{(3)} \right)^2 + \left( (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} (q_{22})^{(3)} G_{22} \\
 & + \left( (\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)} \right) \left( (a_{22})^{(3)} (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (a_{22})^{(3)} (q_{20})^{(3)} G_{20}^* \right) \\
 & \left( (\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(22)} T_{21}^* + (b_{21})^{(3)} s_{(20),(22)} T_{20}^* \} = 0 \\
 & + \\
 & \left( (\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)} \right) \{ (\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)} \} \\
 & \left[ \left( (\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)} \right) (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (q_{24})^{(4)} G_{24}^* \right] \\
 & \left( (\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(25)} T_{25}^* + (b_{25})^{(4)} s_{(24),(25)} T_{25}^* \\
 & + \left( (\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)} \right) (q_{24})^{(4)} G_{24}^* + (a_{24})^{(4)} (q_{25})^{(4)} G_{25}^* \\
 & \left( (\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(24)} T_{25}^* + (b_{25})^{(4)} s_{(24),(24)} T_{24}^* \\
 & \left( (\lambda)^{(4)} \right)^2 + \left( (a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} \\
 & \left( (\lambda)^{(4)} \right)^2 + \left( (b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)} \right) (\lambda)^{(4)} \\
 & + \left( (\lambda)^{(4)} \right)^2 + \left( (a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} (q_{26})^{(4)} G_{26}
 \end{aligned}$$

$$\begin{aligned}
& + ((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}) ((a_{26})^{(4)}(q_{25})^{(4)}G_{25}^* + (a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G_{24}^*) \\
& \left( ((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})s_{(25),(26)}T_{25}^* + (b_{25})^{(4)}s_{(24),(26)}T_{24}^* \right) \} = 0 \\
& + \\
& ((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)}) \{ ((\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)}) \\
& \left[ ((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)})(q_{29})^{(5)}G_{29}^* + (a_{29})^{(5)}(q_{28})^{(5)}G_{28}^* \right] \\
& \left( ((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(29)}T_{29}^* + (b_{29})^{(5)}s_{(28),(29)}T_{29}^* \right) \\
& + \left( ((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)})(q_{28})^{(5)}G_{28}^* + (a_{28})^{(5)}(q_{29})^{(5)}G_{29}^* \right) \\
& \left( ((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(28)}T_{29}^* + (b_{29})^{(5)}s_{(28),(28)}T_{28}^* \right) \\
& \left( ((\lambda)^{(5)})^2 + ((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}) (\lambda)^{(5)} \right) \\
& \left( ((\lambda)^{(5)})^2 + ((b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)}) (\lambda)^{(5)} \right) \\
& + \left( ((\lambda)^{(5)})^2 + ((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}) (\lambda)^{(5)} \right) (q_{30})^{(5)}G_{30} \\
& + ((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}) ((a_{30})^{(5)}(q_{29})^{(5)}G_{29}^* + (a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G_{28}^*) \\
& \left( ((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(30)}T_{29}^* + (b_{29})^{(5)}s_{(28),(30)}T_{28}^* \right) \} = 0 \\
& + \\
& ((\lambda)^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)}) \{ ((\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)}) \\
& \left[ ((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)})(q_{33})^{(6)}G_{33}^* + (a_{33})^{(6)}(q_{32})^{(6)}G_{32}^* \right] \\
& \left( ((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(33)}T_{33}^* + (b_{33})^{(6)}s_{(32),(33)}T_{33}^* \right) \\
& + \left( ((\lambda)^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)})(q_{32})^{(6)}G_{32}^* + (a_{32})^{(6)}(q_{33})^{(6)}G_{33}^* \right) \\
& \left( ((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(32)}T_{33}^* + (b_{33})^{(6)}s_{(32),(32)}T_{32}^* \right) \\
& \left( ((\lambda)^{(6)})^2 + ((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)}) (\lambda)^{(6)} \right) \\
& \left( ((\lambda)^{(6)})^2 + ((b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)}) (\lambda)^{(6)} \right) \\
& + \left( ((\lambda)^{(6)})^2 + ((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)}) (\lambda)^{(6)} \right) (q_{34})^{(6)}G_{34} \\
& + ((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)}) ((a_{34})^{(6)}(q_{33})^{(6)}G_{33}^* + (a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G_{32}^*) \\
& \left( ((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(34)}T_{33}^* + (b_{33})^{(6)}s_{(32),(34)}T_{32}^* \right) \} = 0
\end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

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